

TECHNICAL NOTE

D-1230

ANALYSIS OF TWO THRUSTING TECHNIQUES FOR
SOFT LUNAR LANDINGS STARTING FROM A
50-MILE ALTITUDE CIRCULAR ORBIT

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SUMMARY

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An analytical study has been made of two modes of thrusting to perform soft lunar landings starting from a circular orbit around the moon. One method made use of constant-thrust, restartable engines. In this landing mode a short thrust period is used to initiate the landing maneuver. This is followed by a coasting period, after which thrust is applied to perform the landing. The second landing mode presupposes the use of engines having two levels of thrust. The low thrust level is used to initiate the landing maneuver and is applied until conditions are attained which permit use of the higher thrust level for landing the vehicle. In all instances the thrust vector is directed against the velocity vector.

The results of the study showed that either landing mode could be made quite economical by proper choice of maximum thrust available and the range covered in the landing maneuver. Use of a maximum ratio of thrust to initial earth weight of 0.45 combined with a surface travel of about 30° requires a characteristic velocity of about 6,000 feet per second, which is about 6.5 percent greater than the value of 5,630 feet per second required for a two-impulse Hohmann transfer.

INTRODUCTION

Soft landings of manned spacecraft on the moon are becoming of increasing interest as space exploration proceeds through the preliminary stages. Two general approaches to lunar landings have been under consideration for some time. These approaches are: (1) direct lunar landings, wherein the vehicle approach hyperbolic trajectory intercepts the moon, and (2) the establishment of a lunar parking orbit and a subsequent landing of all or part of the vehicle on the lunar surface. From considerations of safety of the overall mission, and the desirability of close examination of the landing site prior to performing the

landing maneuver, the second approach listed above appears rather attractive and is the approach used in this paper. In the present paper it is assumed that the lunar vehicle is in a close parking orbit around the moon and that the lunar landing will start from this orbit.

There are any number of modes of performing the landing maneuver, and the choice of a maneuver will ultimately depend on factors such as safety, guidance and control requirements, engine characteristics, and so on. Two basic landing modes were investigated in the present study, one in which constant-thrust, restartable engines were assumed and a second in which engines having two levels of thrust were assumed.

SYMBOLS

g_e	acceleration at earth surface due to gravitational attraction, 32.2 ft/sec ²
g_m	acceleration at lunar surface due to gravitational attraction, 5.32 ft/sec ²
h	altitude, ft
I_{sp}	specific impulse, 420 sec
m	mass, slugs
m_0	mass in lunar parking orbit, slugs
r	radial distance from center of moon, ft
\dot{r}	rate of descent, ft/sec
r_m	lunar radius, 5,702,000 ft
T	thrust, lb
t	time, sec
t_f	time during which rocket is firing, sec
V	vehicle velocity referred to inertial axis system, ft/sec
w_0	initial weight in lunar parking orbit (earth weight, $m_0 g_e$), lb

- δ_t angular thrust misalignment, radians
- θ angular travel over lunar surface, deg or radians

Subscripts:

- 1 thrusting period which initiates landing maneuver
- 2 thrusting period which terminates landing maneuver
- i conditions at impact

Dots over symbols denote differentiation with respect to time.

ANALYSIS

Basic Considerations

The most efficient manner of performing a landing from a circular orbit is to apply an impulse of sufficient magnitude to transfer from the circular orbit to an elliptic orbit having its pericynthion at the lunar surface, and then applying a second, larger, impulse at pericynthion to reduce the vehicle velocity to 0. This procedure (Hohmann transfer) is impossible in practice, but can be approximated by use of high-thrust engines. However, it does not appear particularly attractive because of the high-velocity, low-altitude combination at pericynthion and because of the sensitivity of the pericynthion altitude to velocity errors at the end of the first thrusting period. For example, for a circular orbit at a 50-mile altitude the error in pericynthion altitude is about 1,100 feet for each foot-per-second error in radial velocity, and about 4,400 feet for each foot-per-second error in circumferential velocity. The time required for this maneuver is about 1 hour.

A somewhat more reasonable approach is to transfer from the original circular orbit to an elliptic trajectory having a pericynthion at a finite altitude, then perform a braking maneuver to reduce the vehicle velocity to 0 at pericynthion. The vehicle is then permitted to fall freely for a short period, after which thrust is applied to reduce the landing velocity to 0. This total maneuver involves a time period of more than 1 hour, and most of the time is used in the coasting phase from initial retroimpulse until the pericynthion maneuver is initiated. The angular travel over the lunar surface is in excess of 180° . This mode of landing is discussed in detail in reference 1.

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In the present paper, two alternate modes of landing were investigated, one involving the use of a constant-thrust (or constant T/w_0), restartable engine; and the other involving the use of a continuous-operation, variable-thrust engine. The variable-thrust engine was assumed to have two levels of thrust available, a low level for deorbiting and a higher value for the actual landing. The landing modes using these types of engines are explained in the following sections.

Thrusting Modes

If one assumes a specific vehicle in orbit at a given altitude, then there is one value of constant thrust (or constant T/w_0) which can be applied continuously against the velocity vector (gravity turn) to bring the vehicle to the surface with zero velocity. The study reported herein was based on starting from a circular orbit at a 50-mile altitude, and a specific impulse of 420 seconds. The thrust-weight ratio required in this case is $T/w_0 = 0.230$. Continuous application of higher values of thrust will cause the vehicle to attain zero velocity before reaching the surface; however, these engines can be operated intermittently to land with zero velocity. Lower thrust engines will permit the vehicle to impact with finite velocity. A variable-thrust engine could of course utilize a low thrust level to deorbit, and then increase the thrust to land with zero velocity. The study reported herein was made for intermittently operated constant-thrust engines, and for engines having two thrust levels (for convenience, these will be referred to as variable-thrust engines).

Constant-thrust landing mode.- In the constant-thrust landing mode, thrust is applied against the velocity vector to initiate the landing maneuver. After decreasing the vehicle velocity by some finite amount, thrust is terminated and the vehicle continues on a ballistic trajectory. At some point along the trajectory the original thrust level is reapplied until the vehicle reaches the lunar surface with zero velocity. Only two thrusting periods were used in this mode. Figure 1(a) illustrates the constant-thrust landing procedure.

Variable-thrust landing mode.- In the variable-thrust landing mode, low thrust is applied continuously against the velocity vector for a period of time. The thrust level is increased to a higher value somewhere along the trajectory in order to land the vehicle softly (fig. 1(b)). There is of course an infinite choice of combinations of thrust levels to accomplish this mission.

The choice was restricted in this investigation by specifying that only two values of T/w_0 be used for each landing, a maximum value of 0.286 or 0.430 and one lower value. The maximum values chosen correspond to a capsule weight of 14,000 pounds and thrust of 4,000 and

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6,000 pounds, respectively, which are in the range of variable-thrust engines being developed at the Lewis Research Center. The lower thrust levels were selected to be within the range of these same variable-thrust engines. The thrust-weight ratios (T/w_0) assumed for the investigation are given in the following table:

Landing mode	Initial T/w_0	Final T/w_0
Constant thrust	2.000	2.000
	1.000	1.000
	.642	.642
	.430	.430
	.286	.286
	.250	.250
	.230	.230
Variable thrust	0.214	0.286
	.143	.286
	.072	.286
	.043	.286
	.029	.286
	.214	.430
	.143	.430
	.072	.430
	.043	.430
	.029	.430

Equations of Motion

The computations of this investigation were made for a point mass moving in a plane utilizing the following equations of motion

$$\ddot{r} - r\dot{\theta}^2 = -\frac{T}{m} \frac{\dot{r}}{\sqrt{\dot{r}^2 + (r\dot{\theta})^2}} - g_m \left(\frac{r_m}{r}\right)^2 \quad (1)$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = -\frac{T}{m} \frac{r\dot{\theta}}{\sqrt{\dot{r}^2 + (r\dot{\theta})^2}} \quad (2)$$

where

$$m = m_0 + \int \dot{m} dt_f \quad (3)$$

and

$$\dot{m} = - \frac{T}{g_e I_{sp}}$$

As shown by the equations, the thrust is applied against the velocity vector. These equations were solved on an electronic digital computer. An iteration process was used in order to obtain the desired end condition of zero velocity at touchdown. In the coast phases (no thrust applied) the standard orbit equations were used to determine orbit characteristics at various altitudes.

RESULTS AND DISCUSSION

The results of this study are discussed in two groups, the first dealing with the use of constant-thrust engines and the second with the variable-thrust engines.

Constant-Thrust Mode

Trajectory characteristics.— The trajectories for each value of T/w_0 are shown in figure 2 as a curve of altitude plotted against angular range after initiation of the landing maneuver. Generally there are five trajectories shown for each value of T/w_0 , and each trajectory is associated with a different length of the initial thrusting period. The duration of the initial thrusting periods was selected on the following basis: For a given T/w_0 , minimum range is achieved by prolonging the initial thrusting period until the vehicle velocity reaches zero. Shorter periods of initial thrust will extend the range of the landing maneuver. In this study the duration of the initial thrusting periods were fractions of the time required to completely stop the vehicle. A limiting case is reached when $T/w_0 = 0.230$, since continuous thrusting will stop the vehicle on the lunar surface, and hence no coasting period is permissible. The ranges shown in figure 2 are generally less than 1.5 radians (about 90°) for reasons which will be pointed out in later sections.

The sensitivity of range to the initial thrusting period is shown in figure 3. As would be expected, range sensitivity to $t_{f,1}$ decreases as $t_{f,1}$ increases. The reason is, of course, that the trajectory becomes more radial as $t_{f,1}$ is increased. The minimum range for each

value of T/w_0 can be obtained from figure 3 but is replotted for clarity in figure 4.

The angle of approach of the landing vehicle relative to the lunar surface is important close to the surface where lateral travel might be undesirable because of surface irregularities. The final portions of the various trajectories are shown in figure 5. It is seen that as T/w_0 increases the angle of approach decreases for the range of $t_{f,1}$ considered. In addition, for a given value of T/w_0 , as $t_{f,1}$ increases (range decreases) the angle of approach increases. The final approach to touchdown should be a vertical descent in actual application. It should be noted, however, that a very long vertical descent does not appear to be particularly attractive for a constant thrust, restartable engine. Some of the characteristics of the minimum-range trajectories, which result in a vertical approach, are shown in figures 6 and 7. Figure 6 shows the altitudes at start and end of the free-fall period. Figure 7 shows that the rate of descent at the end of free fall is quite high except for engines having T/w_0 close to 0.230. Small errors in engine restart could be serious. This is discussed in the section entitled "Error analysis."

Characteristic velocity.— The characteristic velocity, as used herein, is defined by

$$\Delta V = g_e I_{sp} \log_e \frac{m_0}{m}$$

and is a measure of the fuel consumption required to perform a particular maneuver. The characteristic velocity for the landing maneuver is shown in figure 8 as a function of T/w_0 and angular range over the lunar surface. Two factors of interest are apparent from the curves of figure 8. First, the characteristic velocity is reduced as T/w_0 is increased. Second, the characteristic velocity for a given T/w_0 can be reduced by extending the range. The curve for the impulsive case ($T/w_0 = \infty$) is quite familiar and shows that the minimum ΔV occurs for a range of 180° (Hohmann transfer). It is quite obvious that most of the reduction in ΔV is obtained if the range exceeds about 30° (0.52 radian). Similar trends are obtained using finite-level thrust, however, the actual characteristic velocity for a given range increases as the thrust level decreases. The values of ΔV associated with angular ranges of 30° , 60° , and 90° are shown in figure 9 as a function of T/w_0 . The curves show that the penalty in characteristic velocity associated with using finite thrust level is less than 5 percent (compared to the impulsive case) if T/w_0 exceeds about 0.45.

Time required for landing.— The length of time required for the landing maneuver is shown in figure 10 as a function of thrust level

and initial thrusting period. If one considers the use of a station in a circular orbit about the moon, from which the lunar landing vehicle departs, it is of interest to determine if the complete landing maneuver can be accomplished within sight of the orbiting station. At an orbit altitude of 50 miles the range of visibility measured from the local vertical is an angle of 0.293 radian subtended at the center of the moon. If $\Delta\theta$ is defined as the separation angle between the space station and the landing vehicle at the time of touchdown, then the two will be visible to each other if $\Delta\theta$ is less than 0.293 radian. The values of $\Delta\theta$ are shown in figure 11 as functions of T/w_0 and the total angle of travel. It is seen that the desirable combinations for keeping the lunar landing vehicle within sight of the orbiting station during the landing maneuver consist of long range and high T/w_0 . This is also the combination which results in the lower values of characteristic velocity.

Error analysis.— Various types of errors could be made in attempting to control a landing vehicle in some prescribed manner. For example, the landing mode under consideration requires that engine restart occur at a specific time (or attitude). A delay in engine restart would result in an impact with finite velocity. An early engine restart would result in attainment of zero velocity at a finite altitude, and if thrust were then terminated, the vehicle would fall freely and impact with finite velocity. Several trajectories were computed for early and for late engine restarts. Figure 12 shows the effect of timing error for several values of $t_{f,1}$. Positive values of Δt correspond to late engine restart, whereas negative values of Δt correspond to early engine restart. The results show that the impact velocity can build up quite rapidly with timing errors, particularly for the combination of late restart and high T/w_0 .

Variable-Thrust Mode

Trajectory characteristics.— The trajectories of the two-level, continuous-thrust mode of operation are shown in figure 13 as plots of altitude against angular range after initiation of the landing maneuver. There is only one trajectory compatible with each pair of thrust levels, and there is a specific altitude at which the thrust level must be changed. The trajectories of figure 13 show that, for a given maximum available thrust, range can be increased by a reduction in the initial (low) thrust level. Comparison of figures 13(a) and 13(b) shows that, for a given initial thrust level, the range may be extended by increasing the second (high) thrust level. The sensitivity of range to initial thrust level is shown in figure 14.

The final portion (near touchdown) of each trajectory is shown in figure 15. Generally, the angle of approach is steeper than in the case of the constant-thrust mode (fig. 5).

Characteristic velocity.- The characteristic velocity for the landing maneuver is shown in figure 16 as a function of T_1/w_0 for two values of T_2/w_0 . For a given value of T_2/w_0 , the characteristic velocity shows a small decrease as T_1/w_0 is reduced. Since the range increases as T_1/w_0 decreases (fig. 13), then it follows that the characteristic velocity should decrease slightly as the range is increased. This same trend was noted for the constant-thrust engines (fig. 8). A comparison of characteristic velocities for the two thrust modes is shown in figure 17 as a function of angular range. There are a number of interesting points to note from figures 16 and 17. First, the values of ΔV for the variable-thrust engine fall in the region in which ΔV is relatively insensitive to range. Second, since ΔV is insensitive to T_1/w_0 for a wide range of values of T_1/w_0 , then the range of throttleability need not be unreasonable to make the landing operation economical. For example, from figure 16, assuming $T_2/w_0 = 0.430$, a characteristic velocity of about 5,930 feet per second is obtained with $T_1/w_0 = 0.029$. In this case the range of throttleability is 15 to 1. However, if this range of throttleability is reduced to about 3 to 1, by using a value of $T_1/w_0 = 0.143$, the characteristic velocity is increased to 6,000 feet per second. Thus, little is lost in characteristic velocity by accepting a reasonably low range of throttleability.

Time required for landing.- The length of time required to perform the landing maneuver is shown in figure 18 as a function of T_1/w_0 and T_2/w_0 . For a given value of T_2/w_0 , the required time to land increases as T_1/w_0 is reduced. Also, for a given value of T_1/w_0 , the time to land generally decreases as T_2/w_0 is increased. The separation angle between an orbiting station and the lunar landing vehicle at touchdown is shown in figure 19. For the two-level thrust mode of operation the landing vehicle and the station are within sight of each other at the time of impact of the landing vehicle for the range of T_1/w_0 and T_2/w_0 investigated.

Error analysis.- Two types of possible errors were investigated in the variable-thrust mode. One was an error in time of switching to the high thrust level, and the other was the effect of thrust misalignment. The results are shown in figures 20 and 21, respectively. The computations were made only for the trajectories with $T_2/w_0 = 0.286$ to indicate trends. The value of initial T_1/w_0 does not have a large effect on impact velocity in the range of T_1/w_0 from 0.029 to about 0.143. However, further increase in T_1/w_0 causes a reduction in impact velocity for a given time error. The reason for this effect is that as

T_1/w_0 approaches 0.230, the altitude at which T_2/w_0 is applied is being reduced, and the approach velocity at the time of thrust level change is also being reduced. With T_1/w_0 equal to 0.230, it would not be necessary to apply the second thrust level; that is, $T_1/w_0 = 0.230$ is the thrust level which would be applied continuously from orbit to make the vehicle impact with zero velocity.

The effect of thrust misalignment (which is assumed to exist throughout each trajectory) is shown in figure 21. The results shown are for thrust misalignment in such a direction as to increase the rate of descent. The results show no appreciable effect of T_1/w_0 on the impact error due to thrust misalignment. However, the impact velocity can be quite large for a small misalignment. For example, an angular misalignment of $\frac{1}{2}^\circ$ (0.0087 radian) shows an impact velocity of approximately 420 feet per second for the range of T_1/w_0 investigated.

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CONCLUSIONS

An analytical investigation has been made of two modes of braking from a 50-mile altitude parking orbit and landing on the moon. One method made use of constant-thrust, restartable engines. In this mode a short thrust period is used to initiate the landing maneuver. This thrust period is followed by a coasting period, after which thrust is applied to perform the landing. The second method presupposes the use of engines having two levels of thrust. The low level is used to initiate the landing maneuver. This level is maintained until conditions are reached which permit the second (higher) thrust level to be used to land the vehicle. In all instances the thrust vector is directed against the velocity vector. The following observations were noted from the results of the investigation:

1. The characteristic velocity (or fuel consumption) required to perform the landing maneuver is a function of the maximum thrust level and of the total range of the landing maneuver. In this respect the characteristic velocities of the two landing modes are approximately equal if the range and maximum thrusts are equal.

2. The characteristic velocity, for a given maximum thrust, varies inversely with the range traveled and approaches a minimum as the range is extended. A point of diminishing returns is reached after extending the range to about 30° . Further increase in range will reduce the characteristic velocity by less than 5 percent.

3. For a given desired range, the characteristic velocity can be reduced by increasing the maximum thrust level. Here again a point of diminishing returns is reached at a thrust-weight ratio (earth weight) of about 0.45. A combination of at least 30° of travel and a thrust-weight ratio of 0.45 can be used to perform landings with a characteristic velocity of only about 5 percent in excess of that required for a two-impulse Hohmann transfer. This range can be attained with the constant-thrust engine by adjusting the length of the initial thrust period, and can be obtained with a two-thrust engine if 5 to 1 throttleability is available.

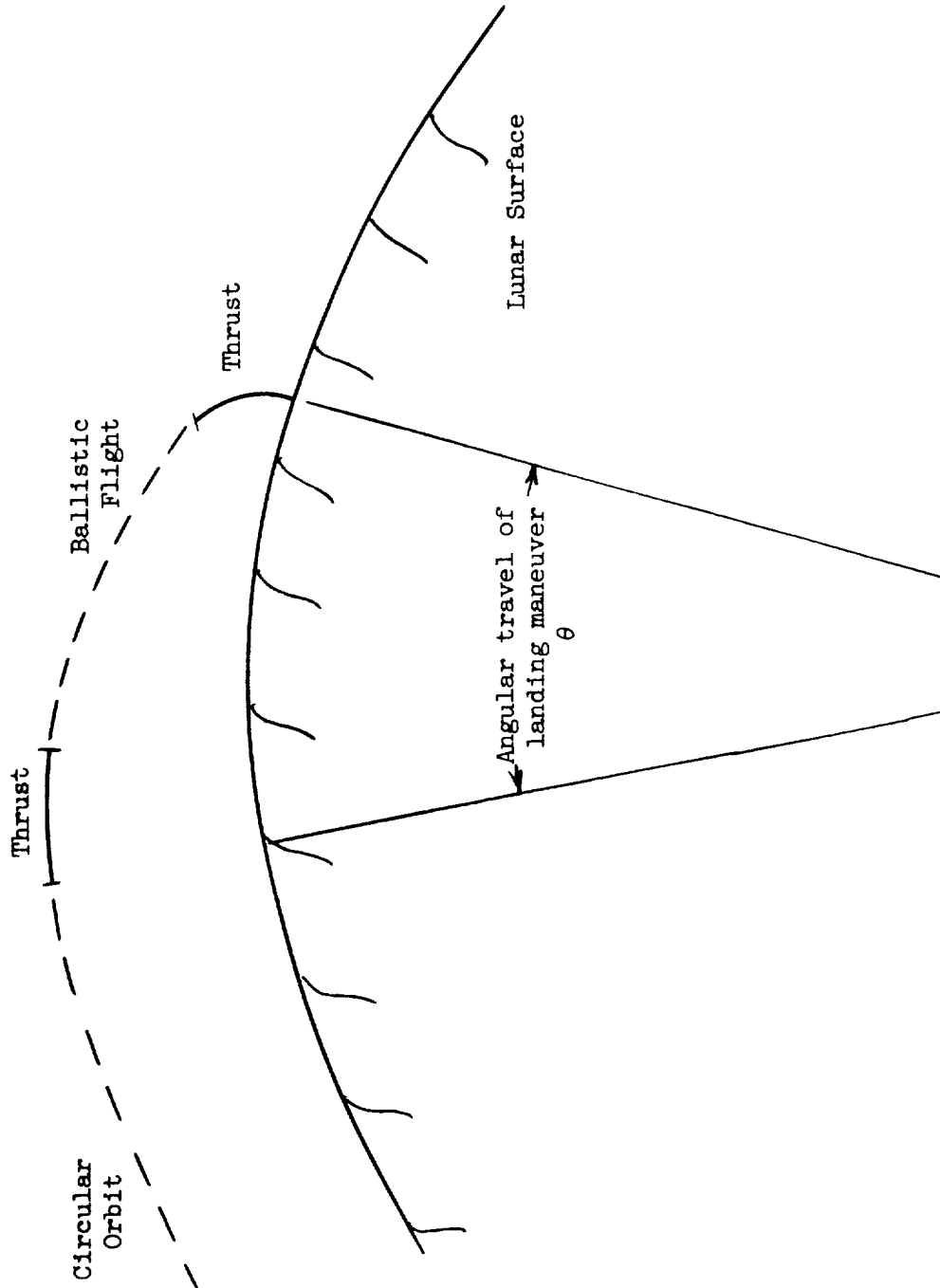
4. It is of interest to note that use of an engine having a maximum thrust-weight ratio of 0.430 and only 3 to 1 throttleability requires a characteristic velocity of $6\frac{1}{2}$ percent in excess of that required for a two-impulse Hohmann transfer. The 3 to 1 range of throttleability is significant in that it is attainable with hydrogen-oxygen engines.

5. If one considers that the lunar landing is to be made by a small vehicle originally in orbit as part of a larger vehicle, it would be desirable that the larger vehicle be within sight with the landing craft at touchdown. The results of the investigation show that this can be accomplished with the landing modes investigated for a wide range of thrust-weight ratios and for a wide spread in range traveled during the landing maneuver.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Air Force Base, Va., January 16, 1962.

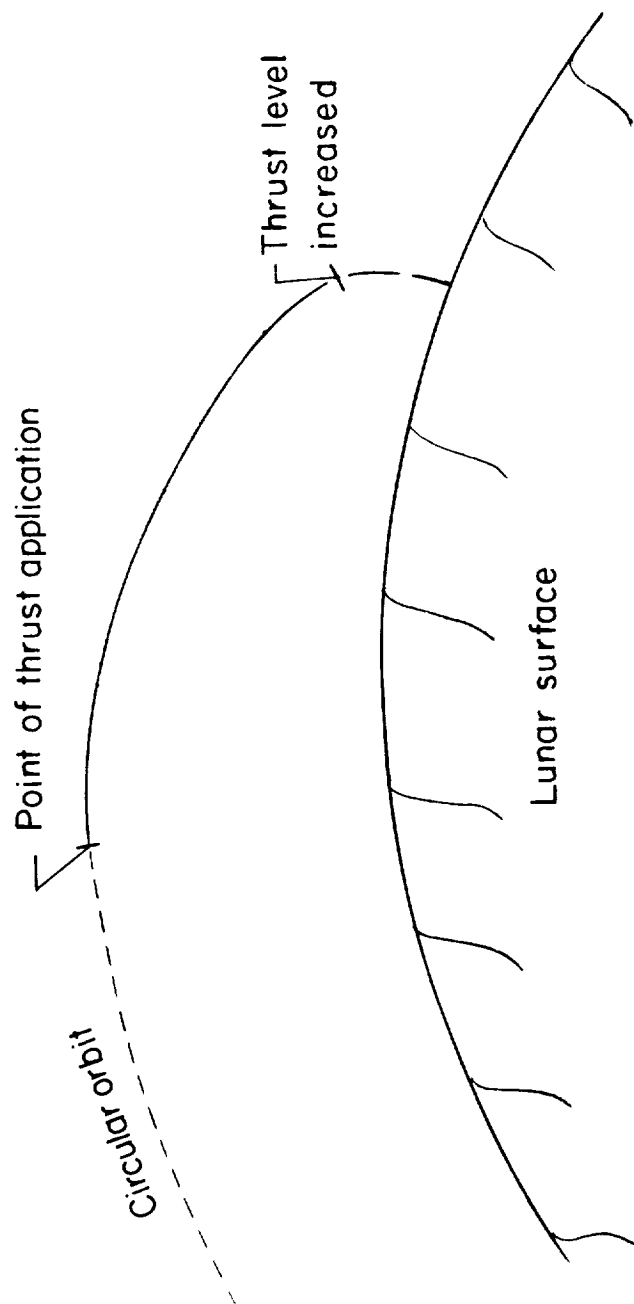
REFERENCE

1. Weber, Richard J., and Pauson, Werner M.: Some Thrust and Trajectory Considerations for Lunar Landings. NASA TN D-134, 1959.



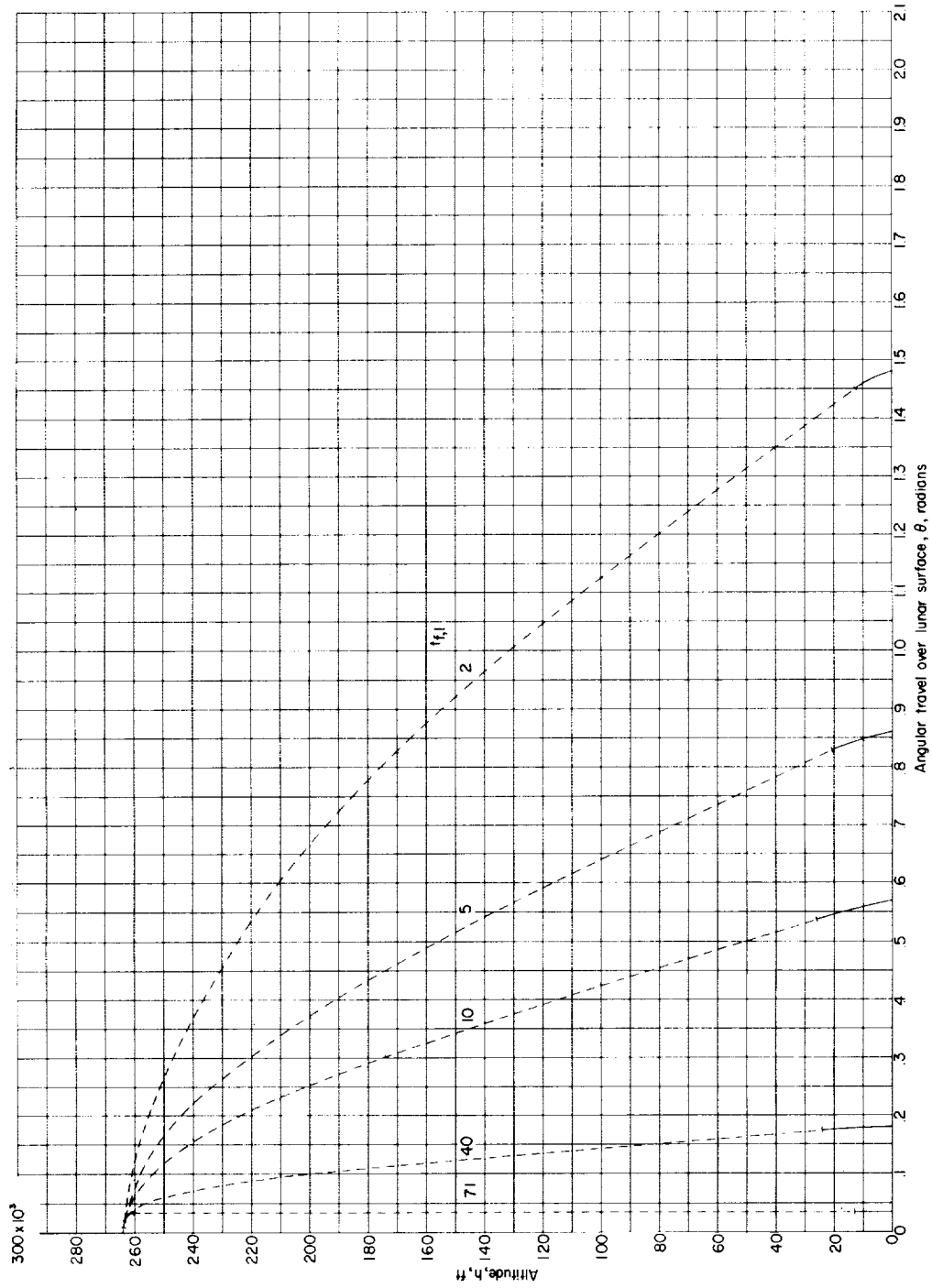
(a) Constant-thrust landing mode.

Figure 1.- Illustration of thrust modes assumed in this investigation.



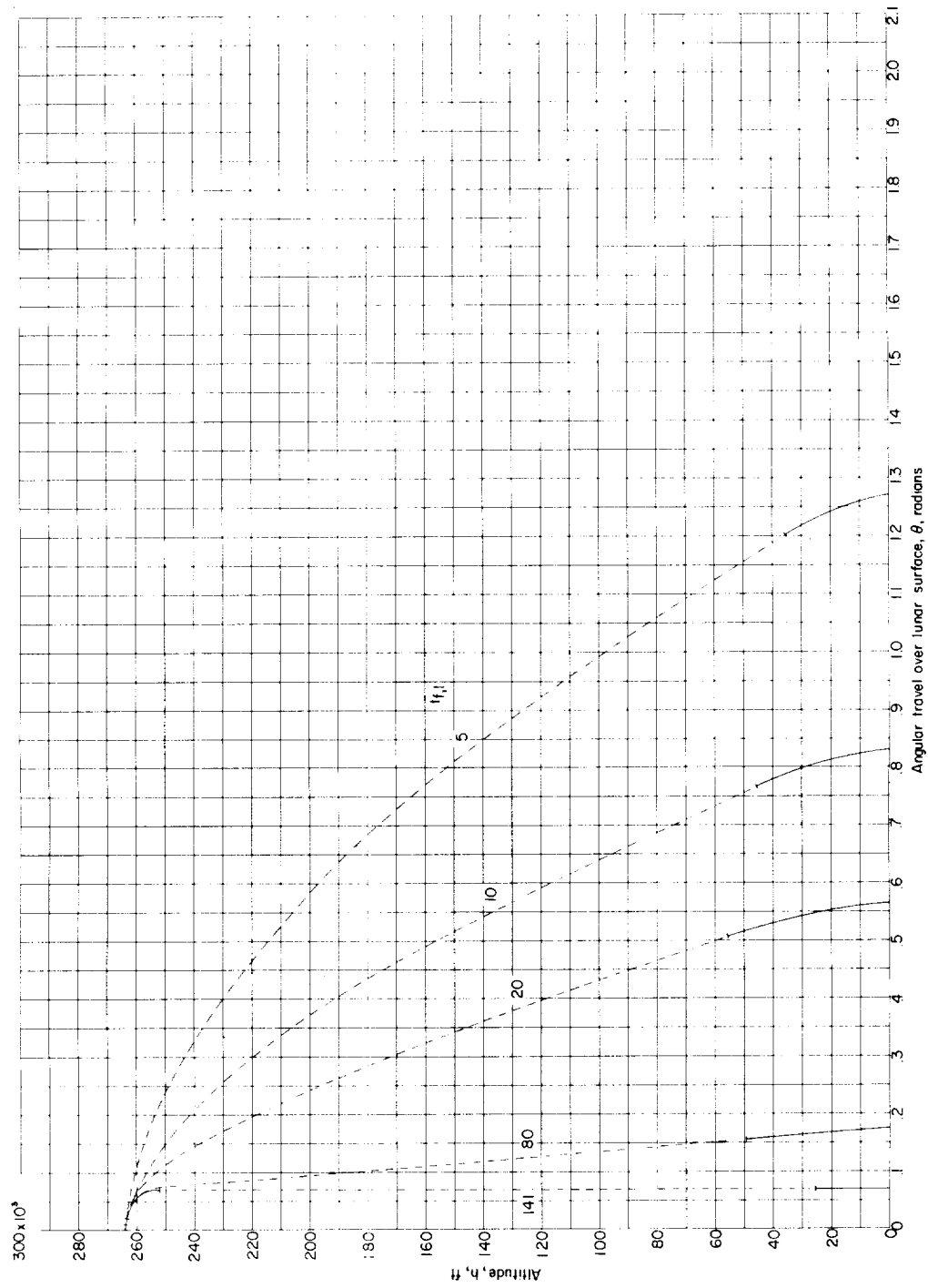
(b) Variable-thrust landing mode.

Figure 1.- Concluded.



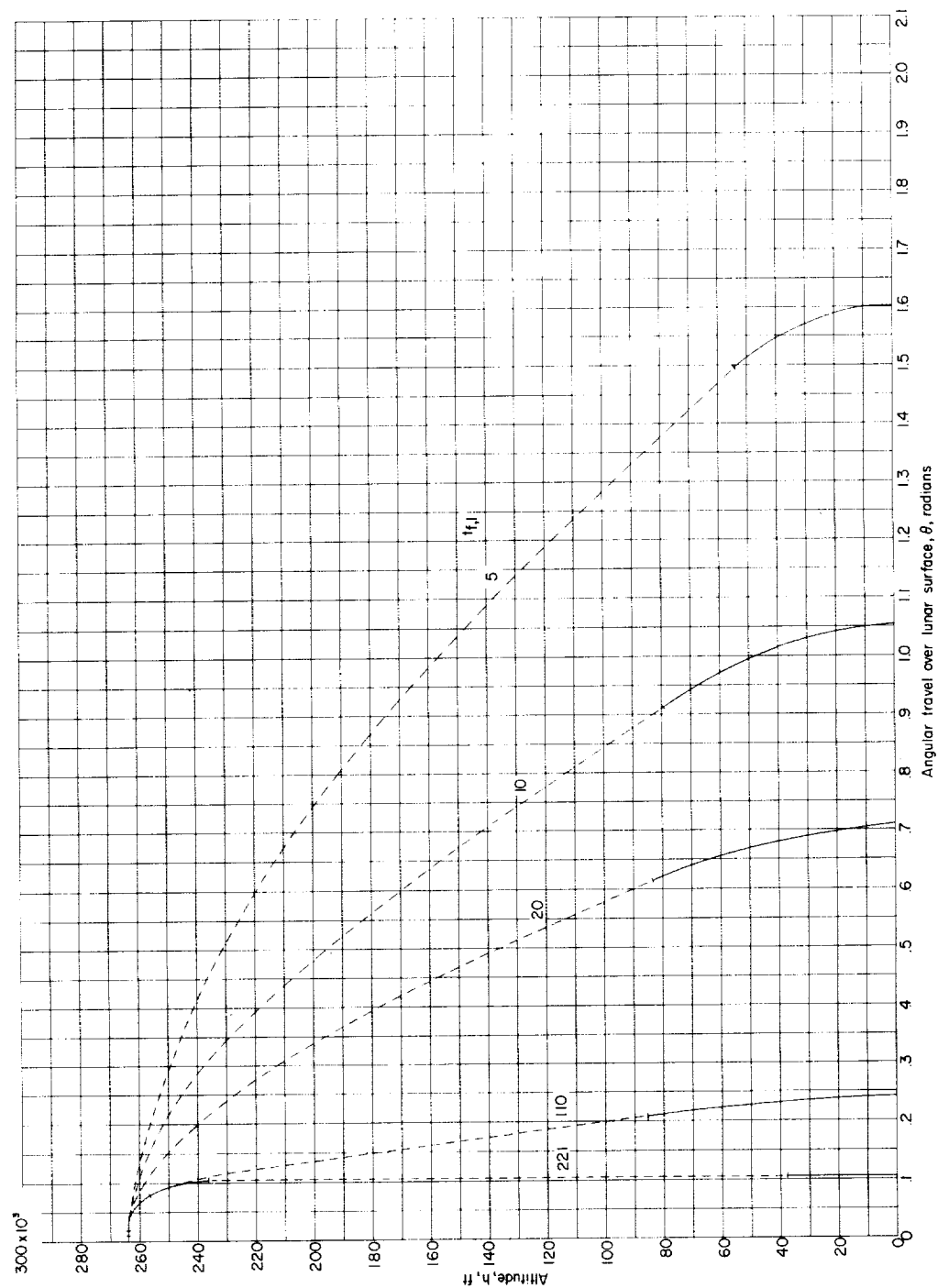
(a) $T/W_0 = 2.0$.

Figure 2.- Trajectory characteristics. Constant-thrust mode.



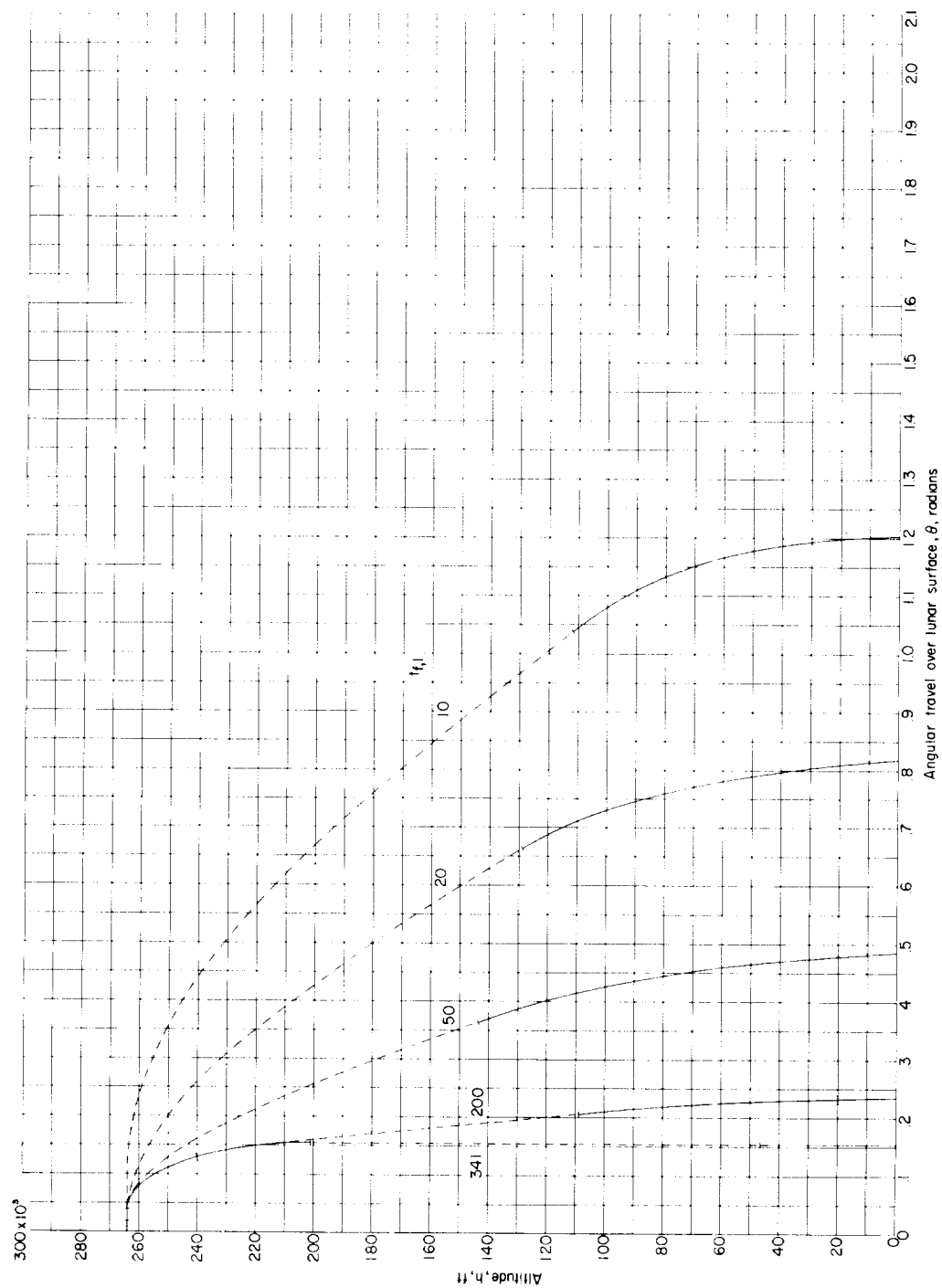
(b) $T/W_0 = 1.0$.

Figure 2.- Continued.



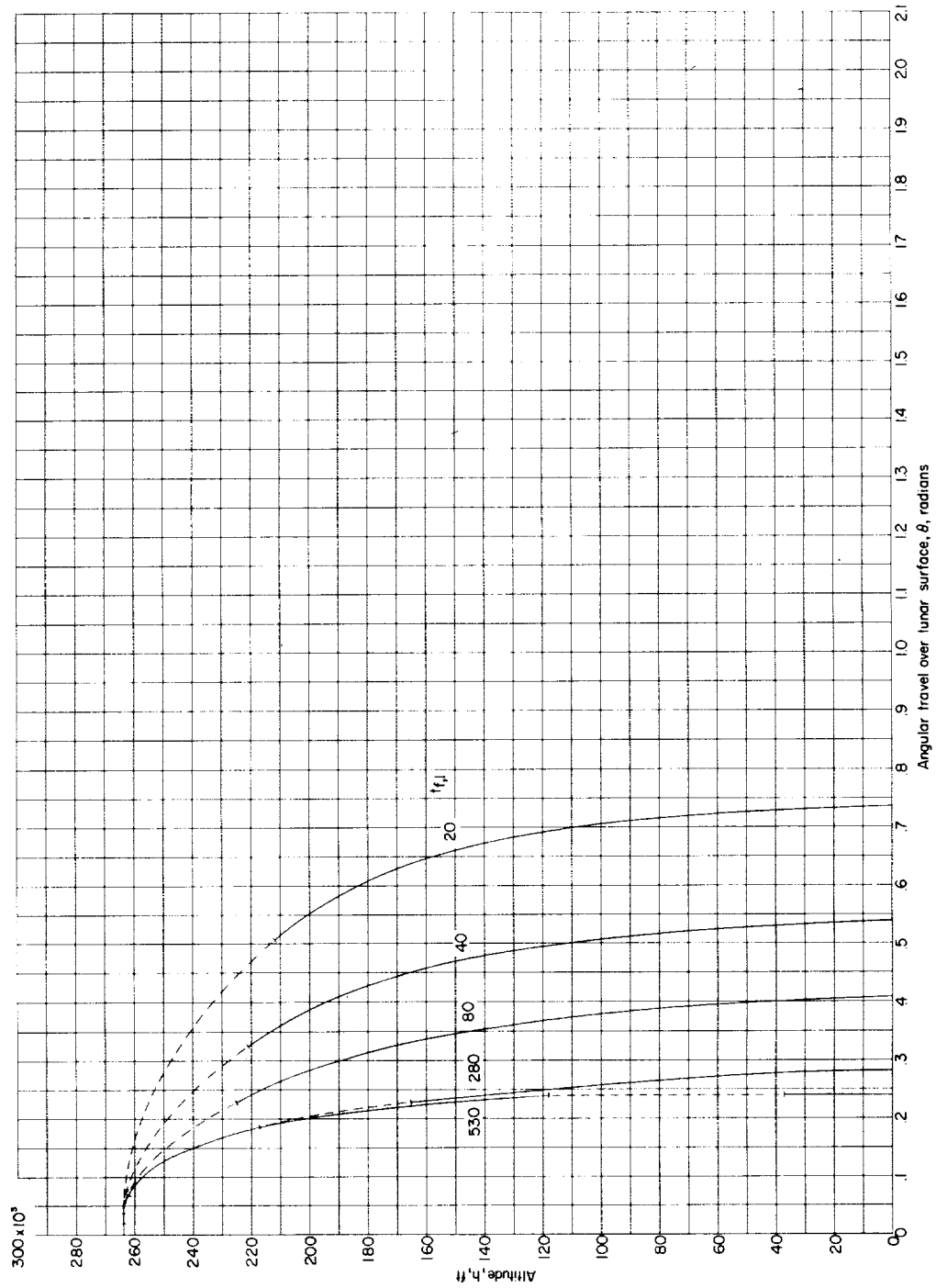
(c) $T/w_0 = 0.642$.

Figure 2.- Continued.



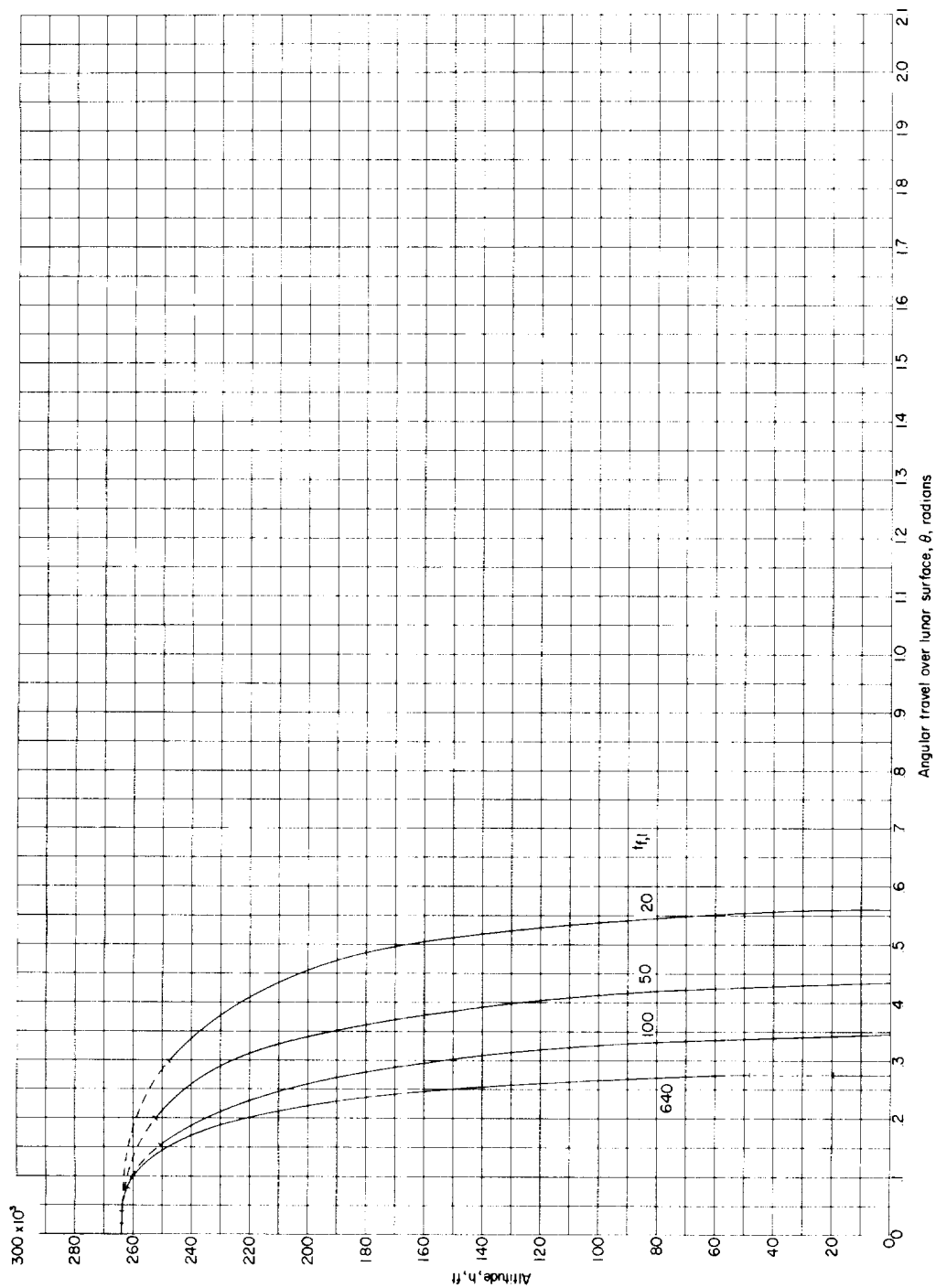
(d) $T/W_0 = 0.430$.

Figure 2.- Continued.



(e) $T/w_0 = 0.286$.

Figure 2.- Continued.



(f) $T/w_0 = 0.250$.

Figure 2.- Concluded.

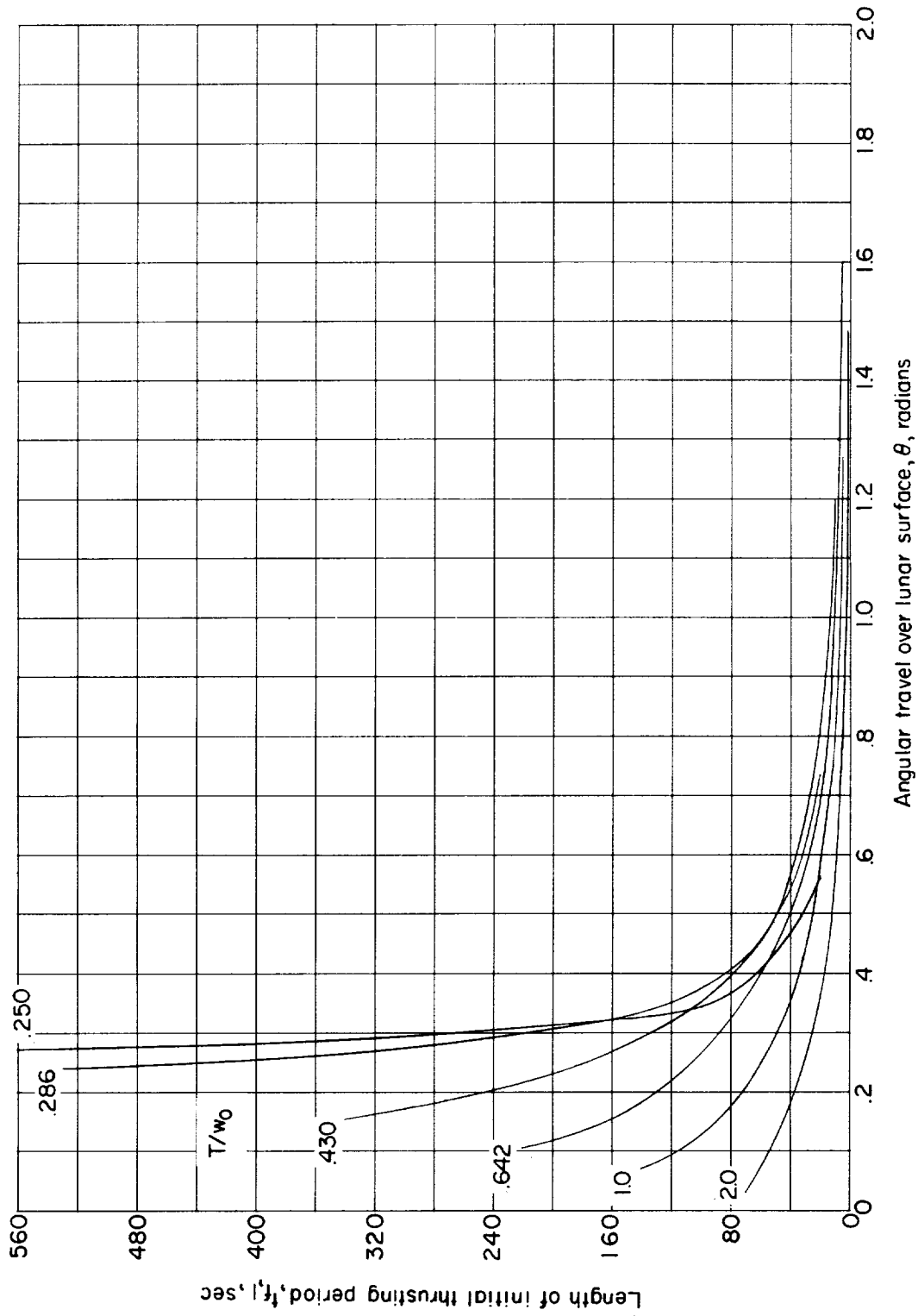


Figure 3.- Sensitivity of range to duration of initial thrusting period. The angular travel is from time of first thrust initiation until touchdown.

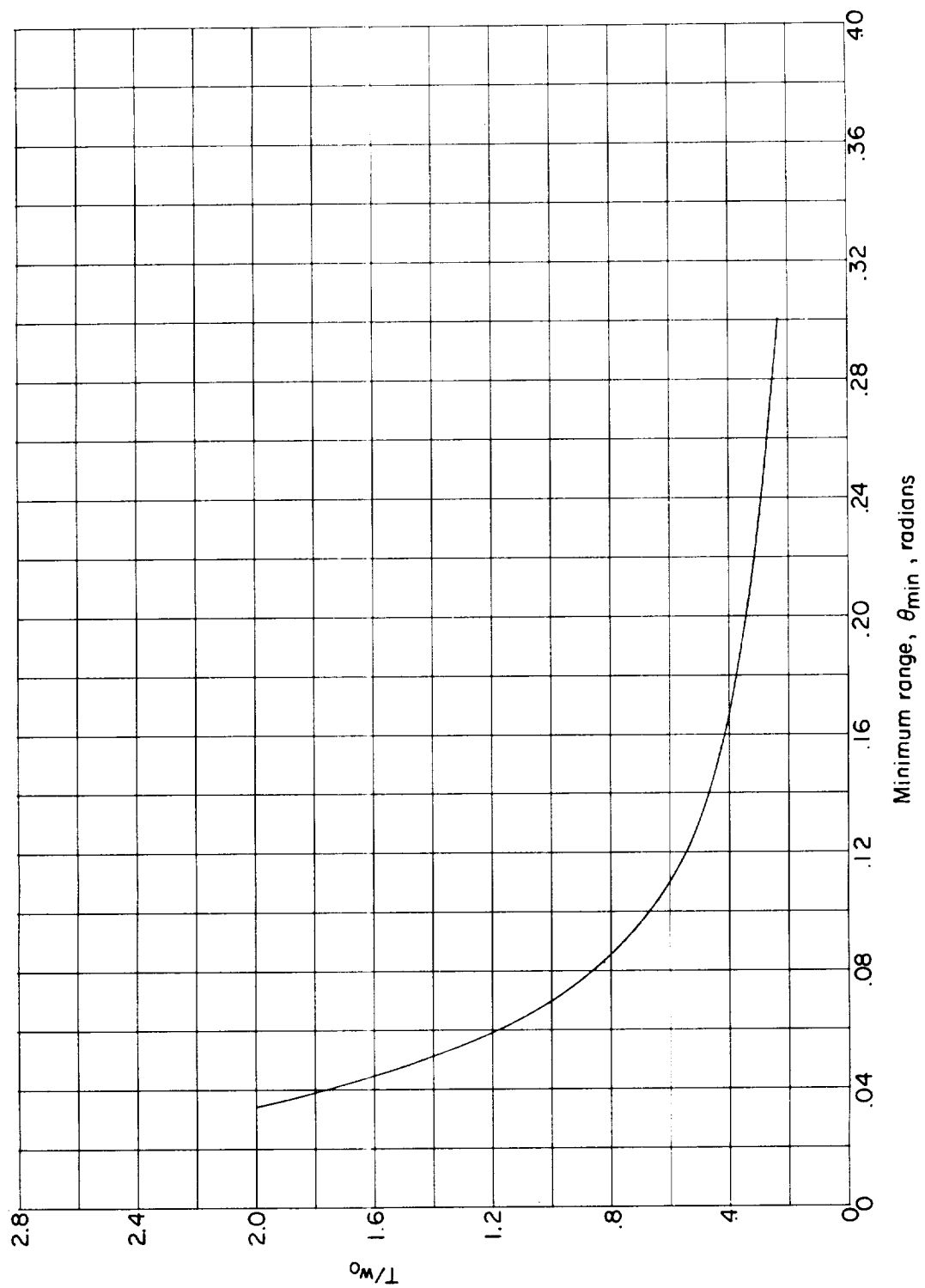
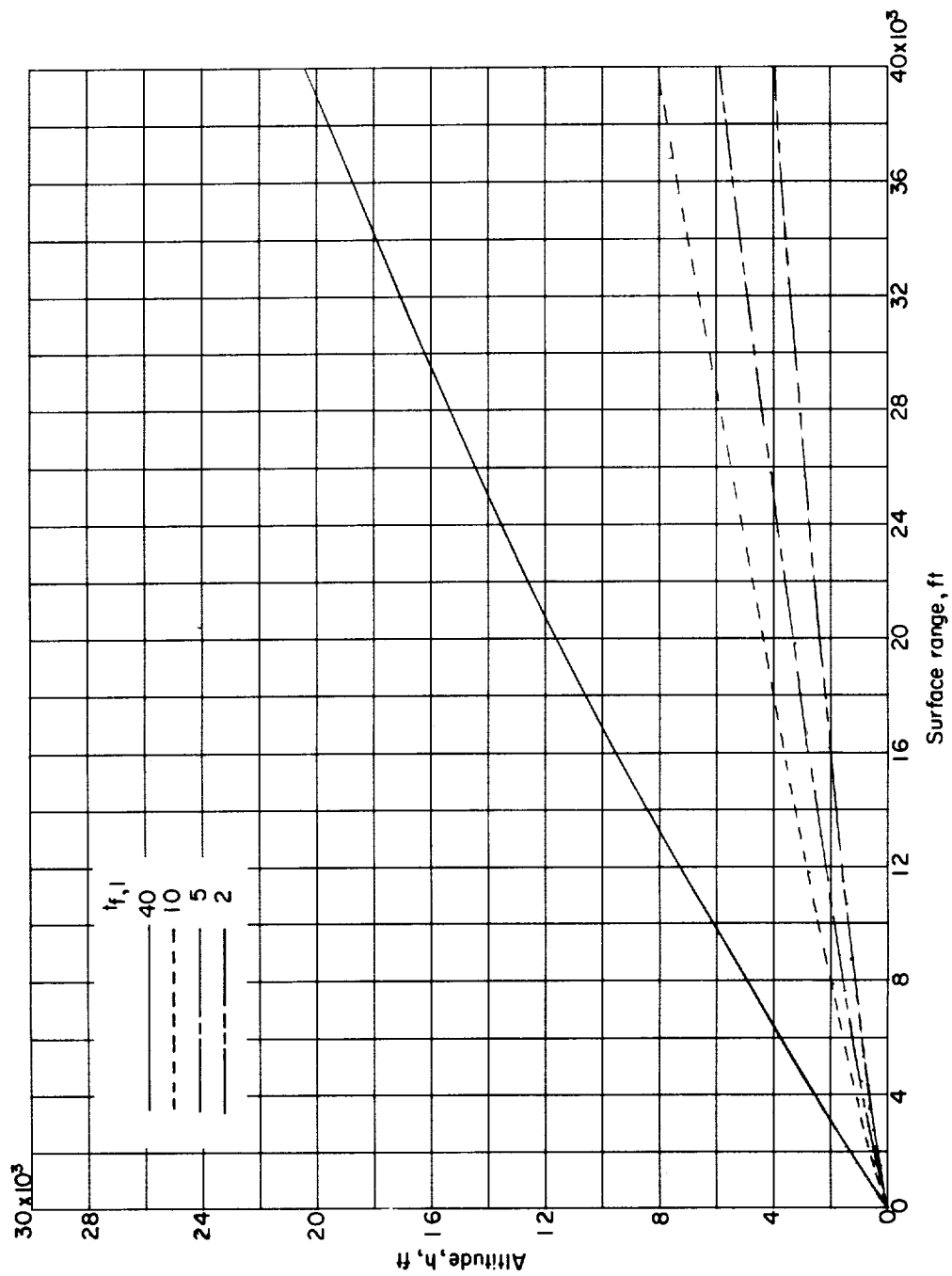
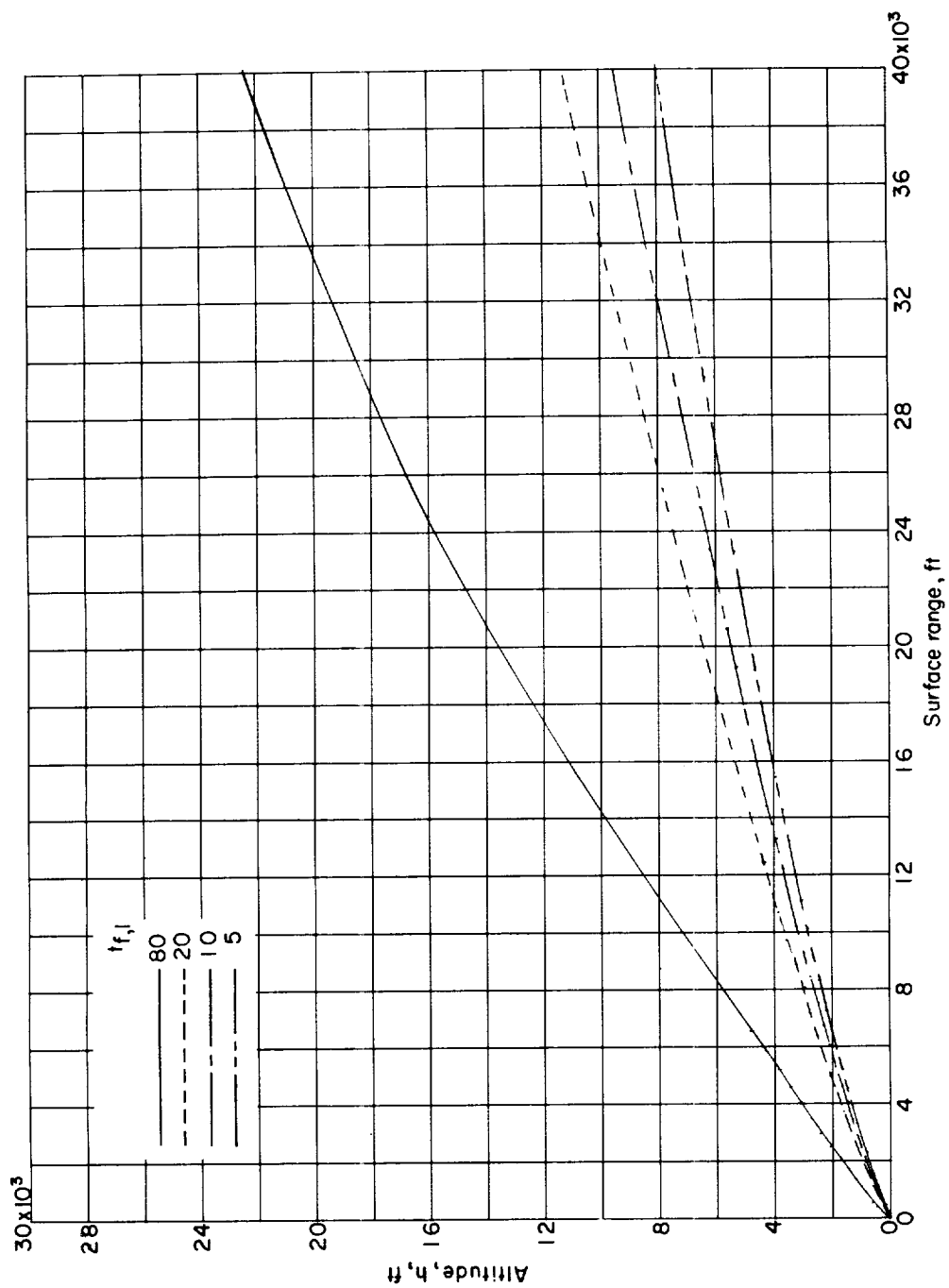


Figure 4.- Minimum range obtainable as a function of T/w_0 .



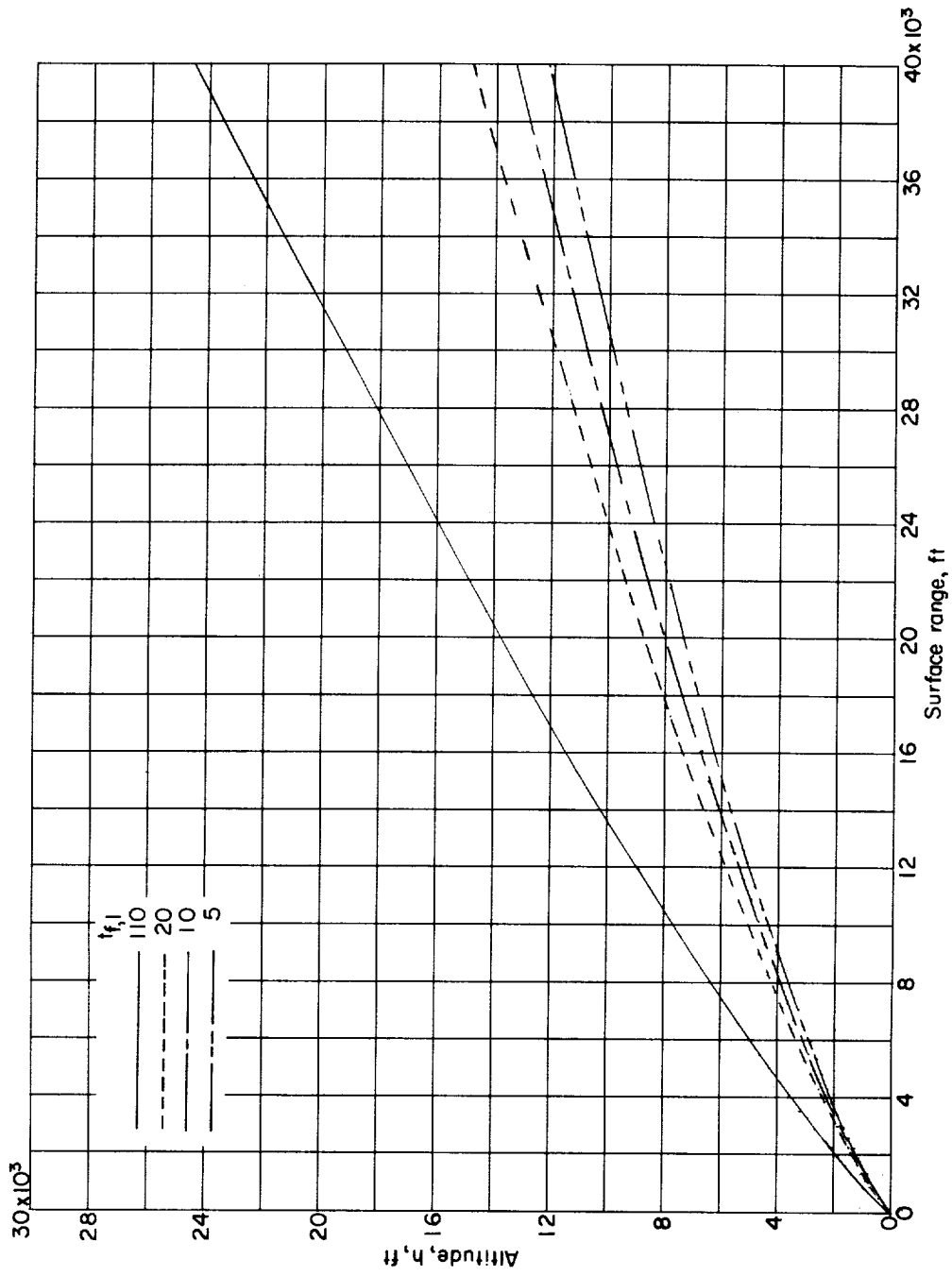
(a) $T/w_0 = 2.0$.

Figure 5.- Trajectory characteristics near impact point.



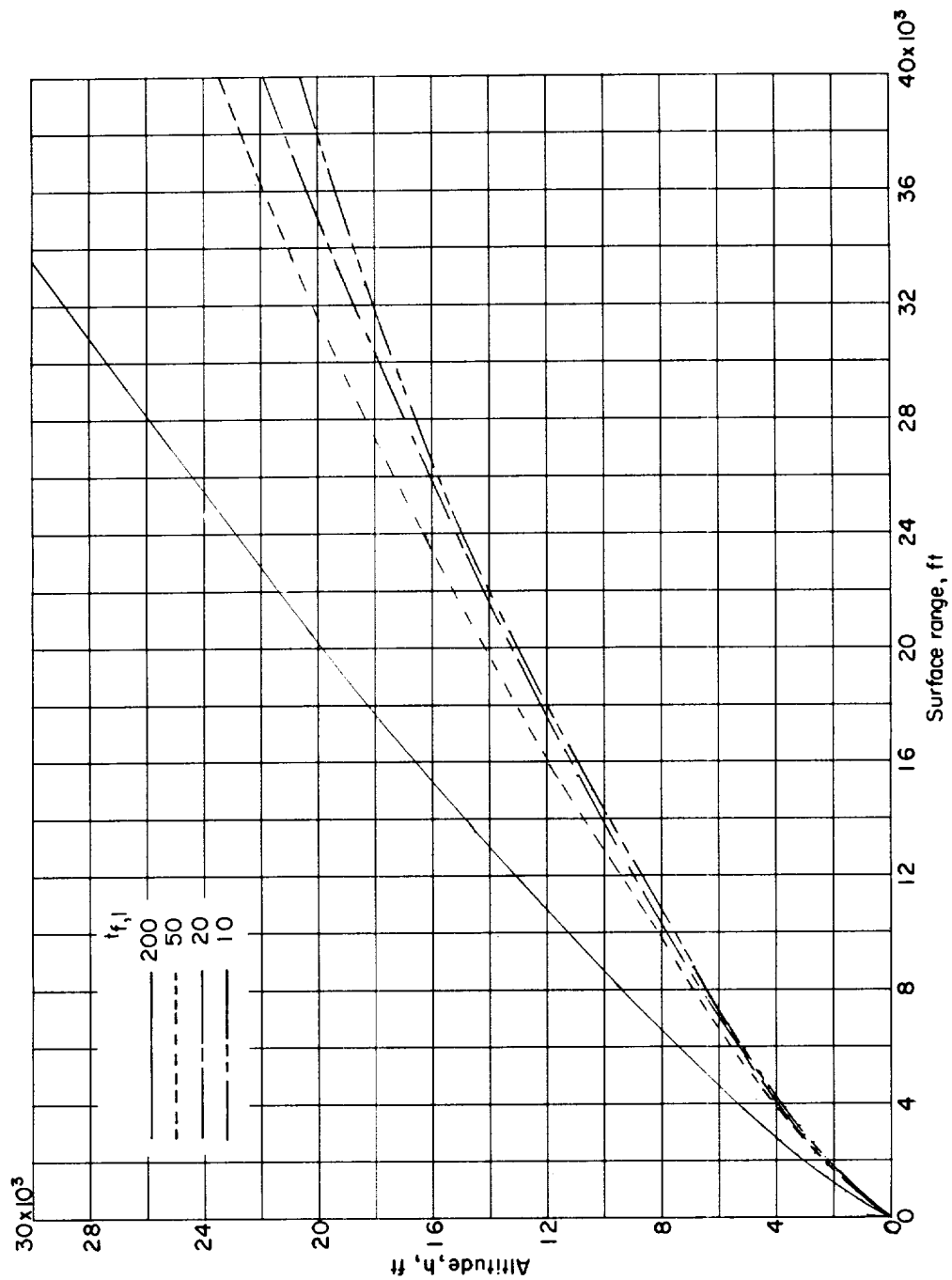
(b) $T/w_0 = 1.0$.

Figure 5.- Continued.



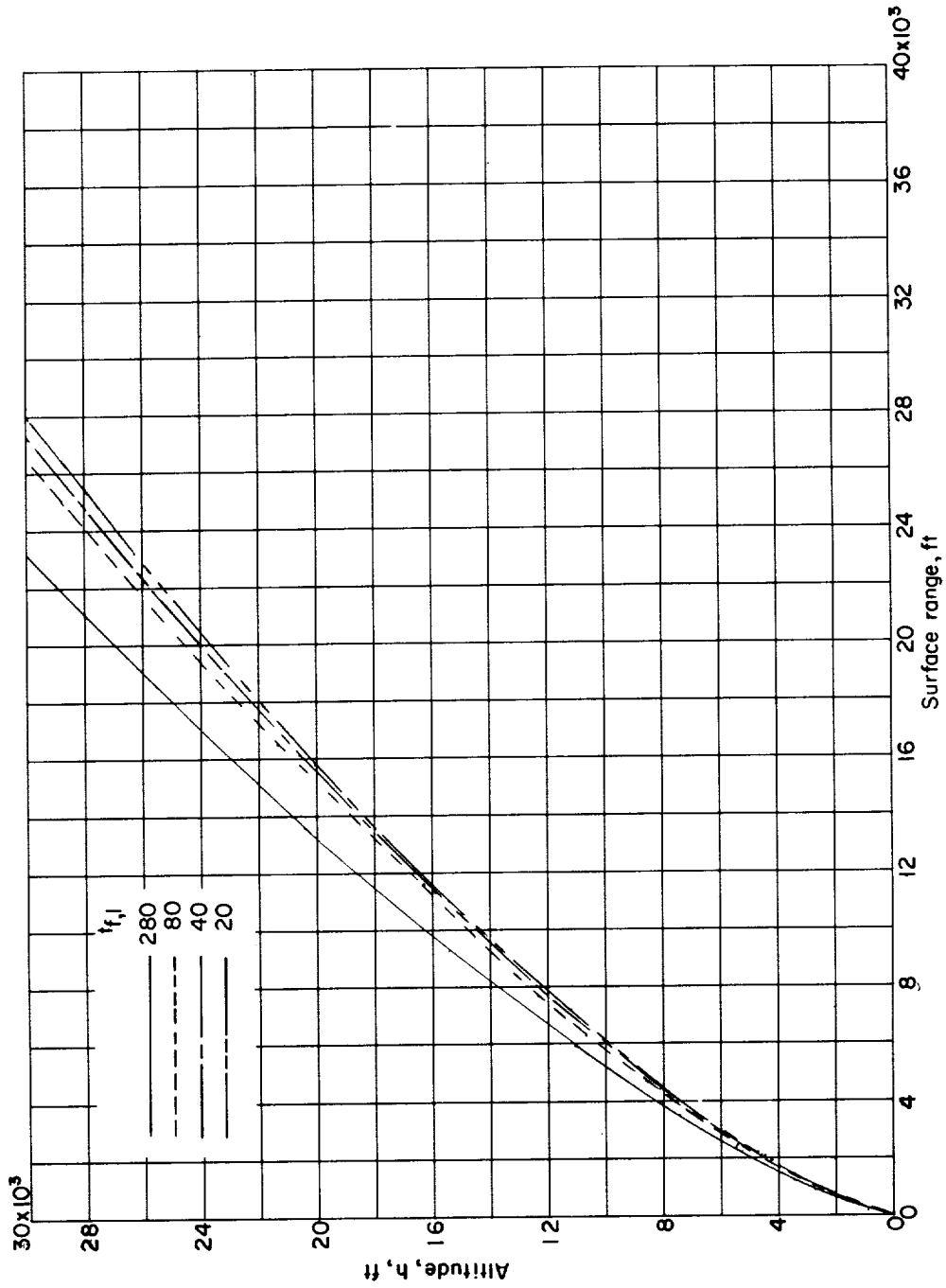
(c) $T/w_0 = 0.642$.

Figure 5.- Continued.



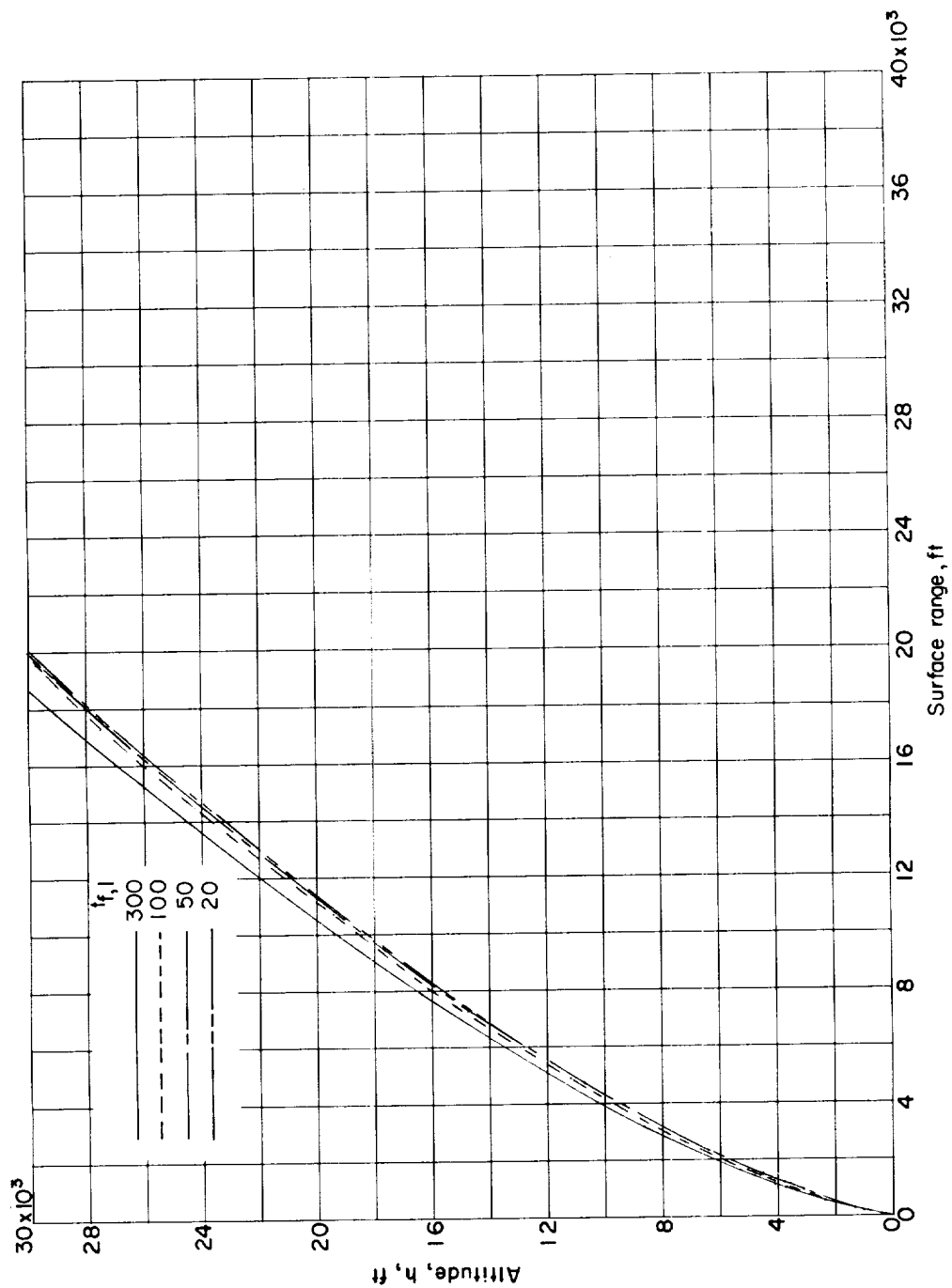
(d) $T/w_0 = 0.430$.

Figure 5.- Continued.



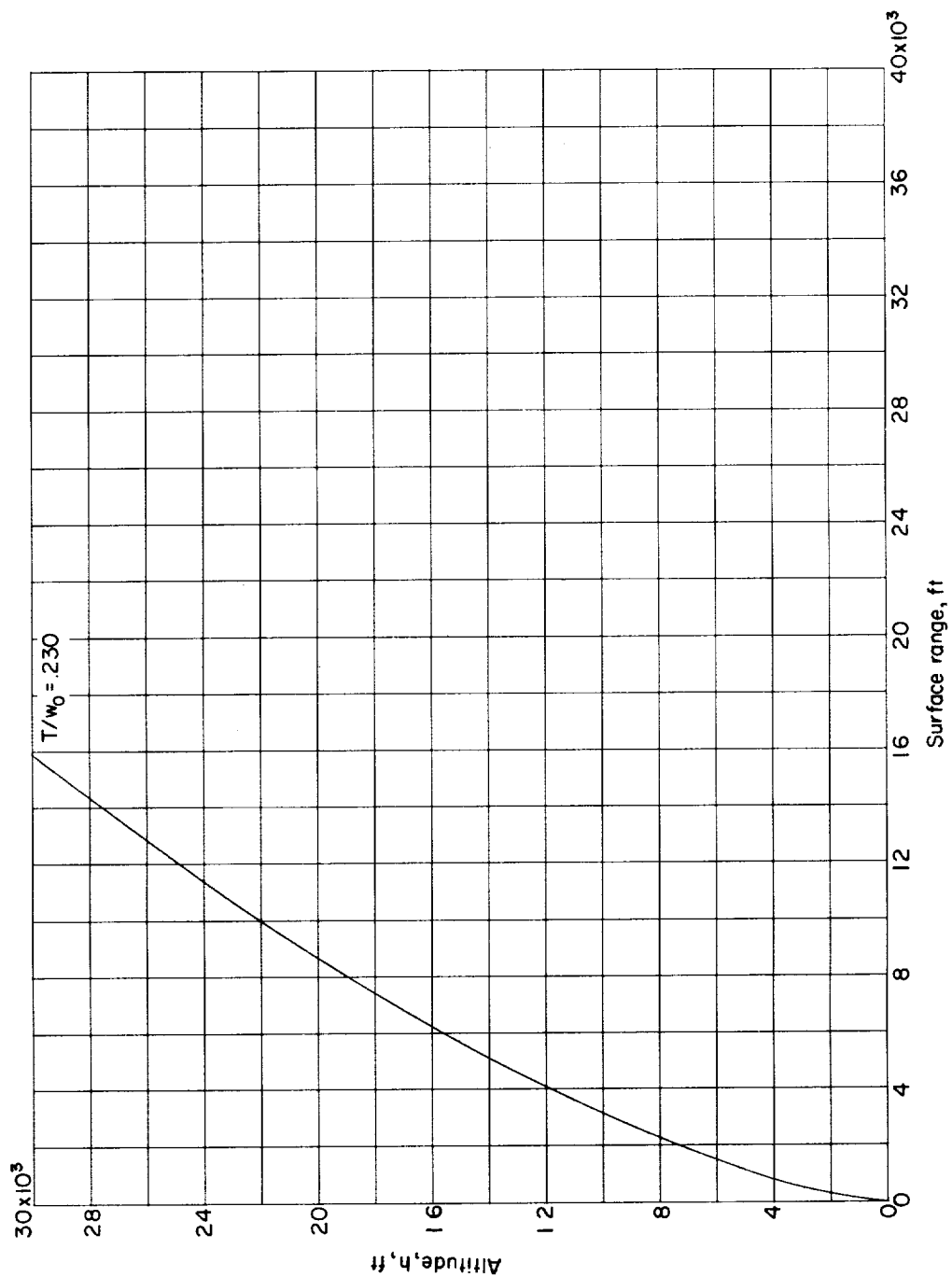
(e) $T/w_0 = 0.286$.

Figure 5.- Continued.



(f) $T/w_0 = 0.250$.

Figure 5.- Continued.



(g) $T_1/w_0 = 0.230$.

Figure 5.- Concluded.

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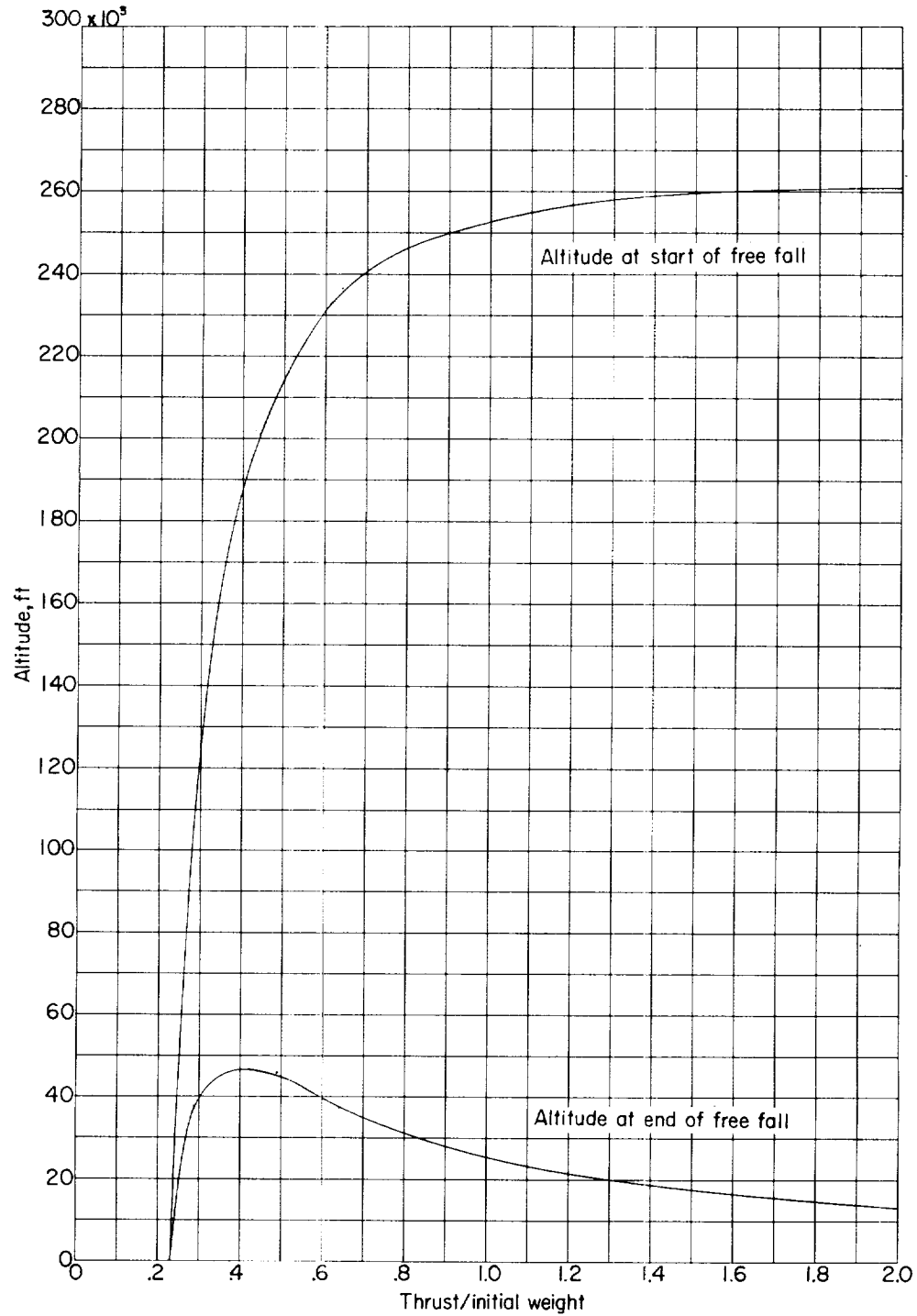


Figure 6.- Altitude for start and end of free-fall period for minimum range trajectory.

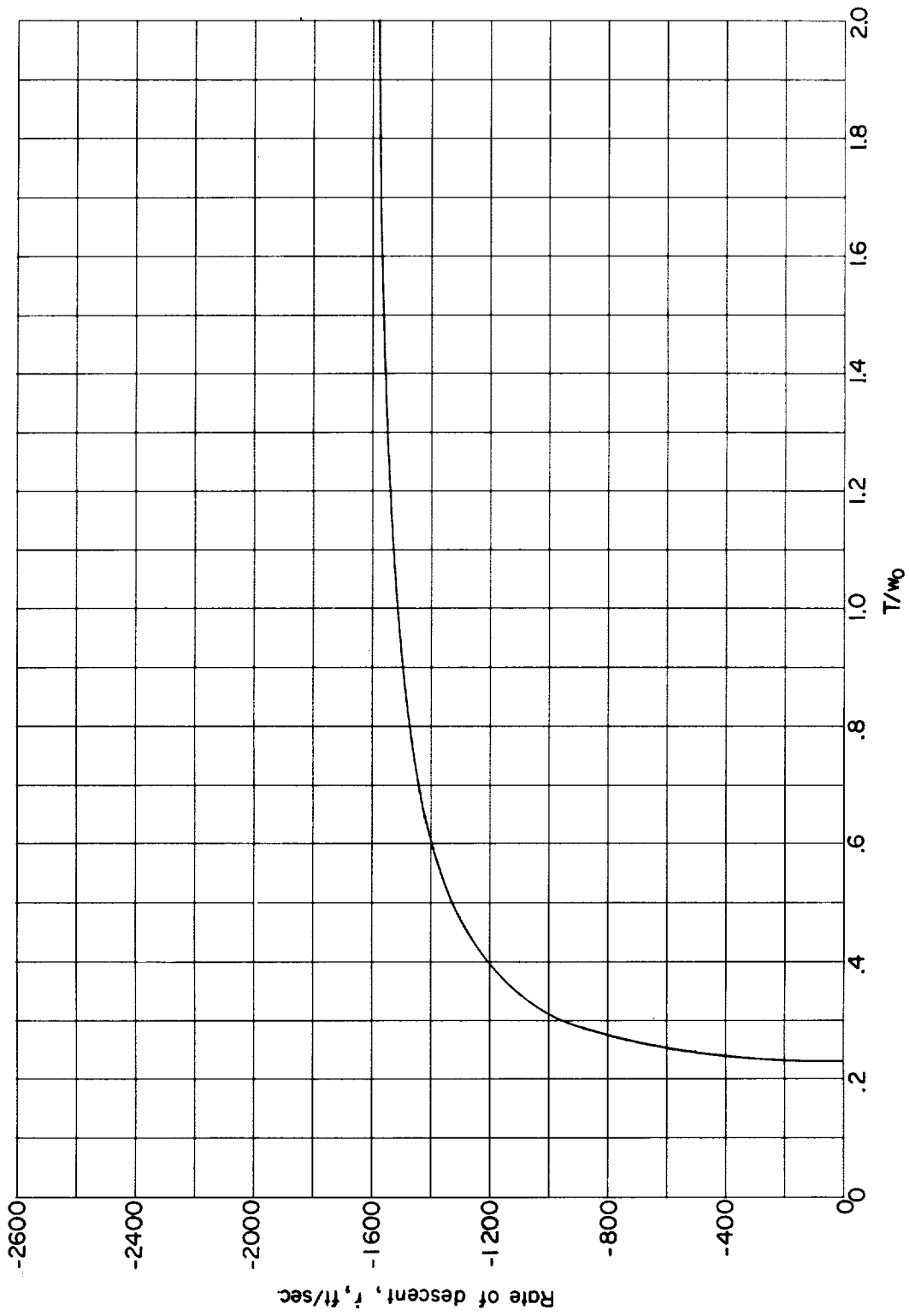


Figure 7.- Vertical velocity at end of free-fall period (or engine restart altitude) for minimum range trajectory.

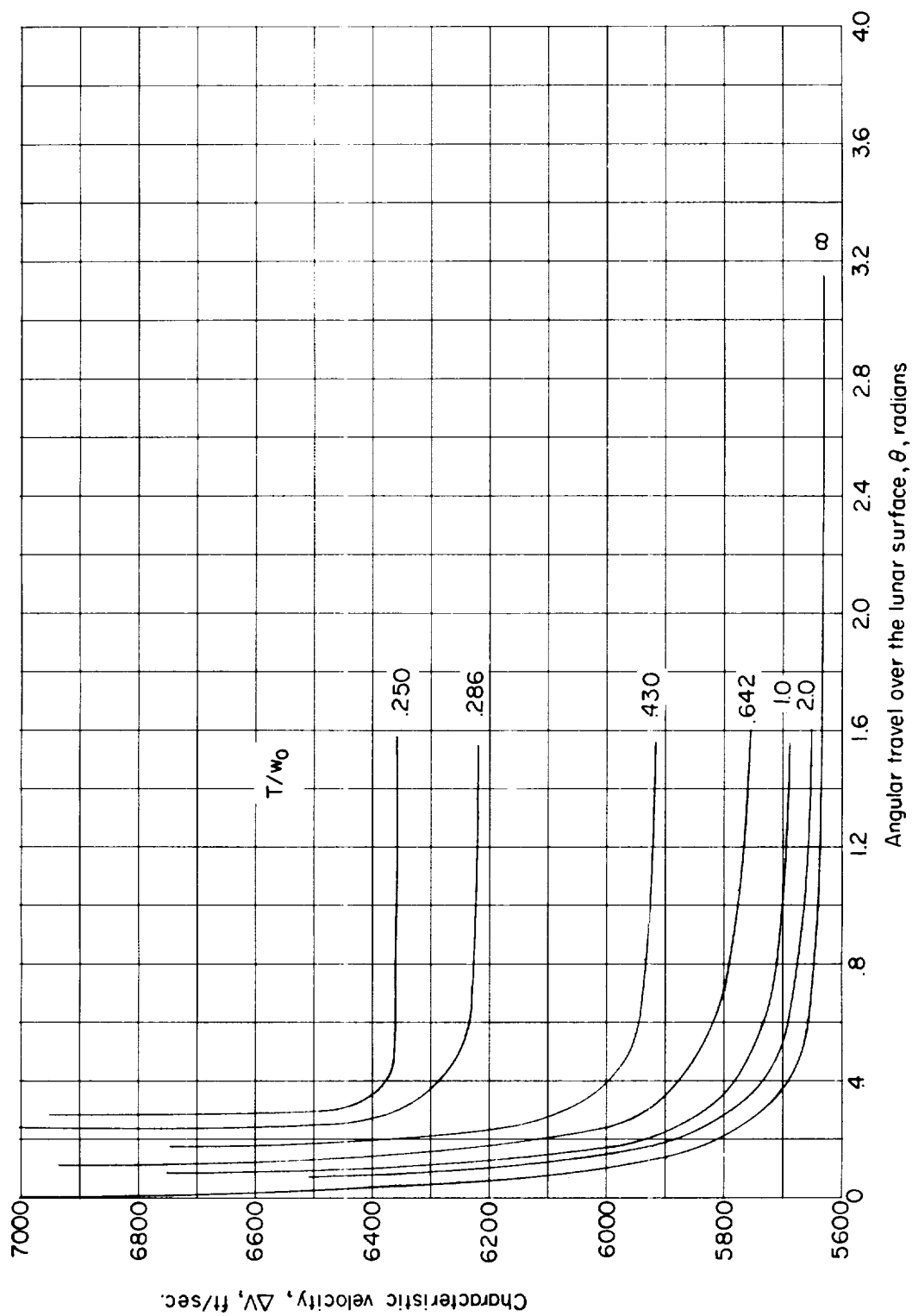


Figure 8.- Characteristic velocity as a function of T/w_0 and angular travel over the lunar surface.

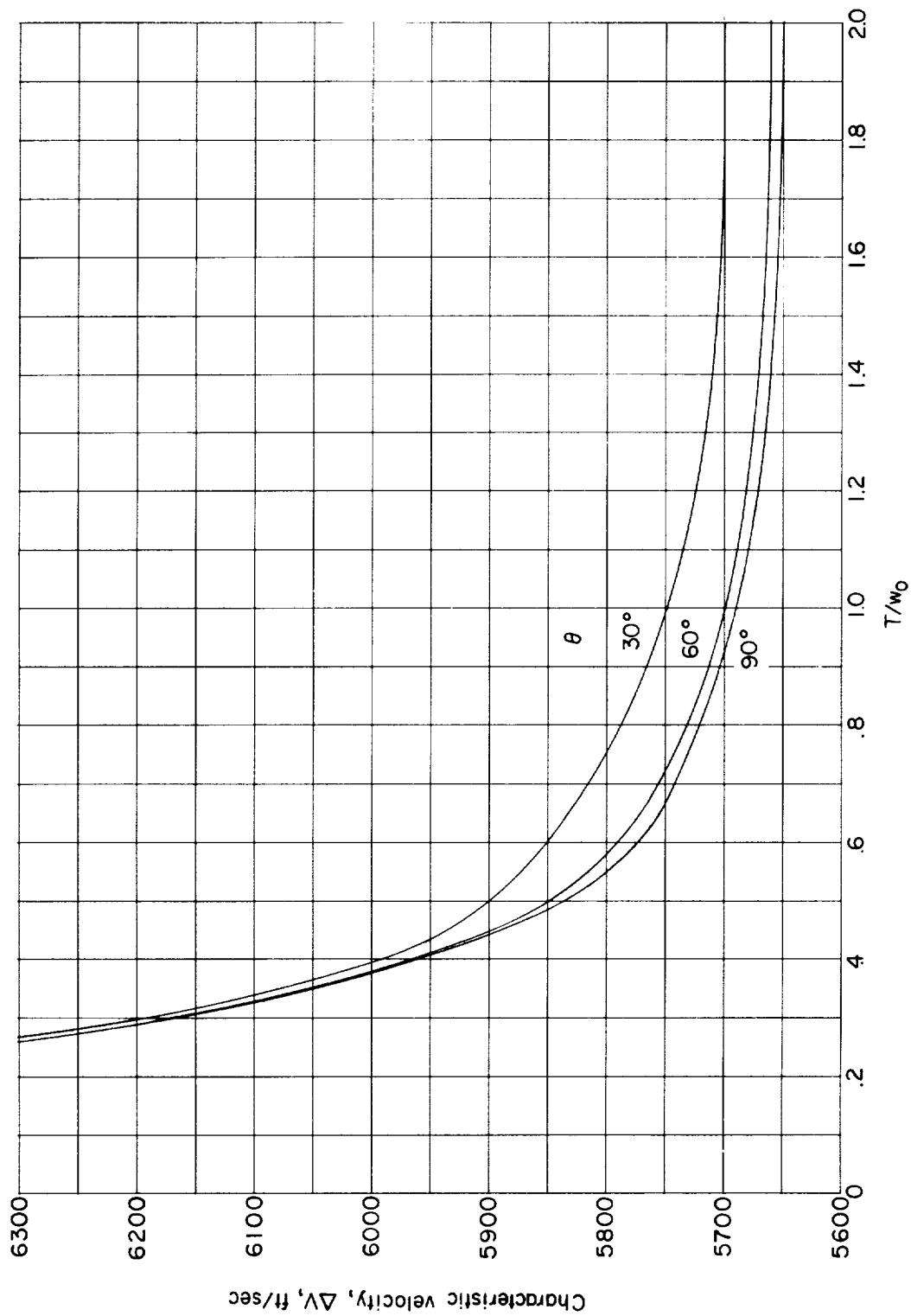


Figure 9.- Characteristic velocity associated with angular travel θ over the lunar surface as a function of T/w_0 .

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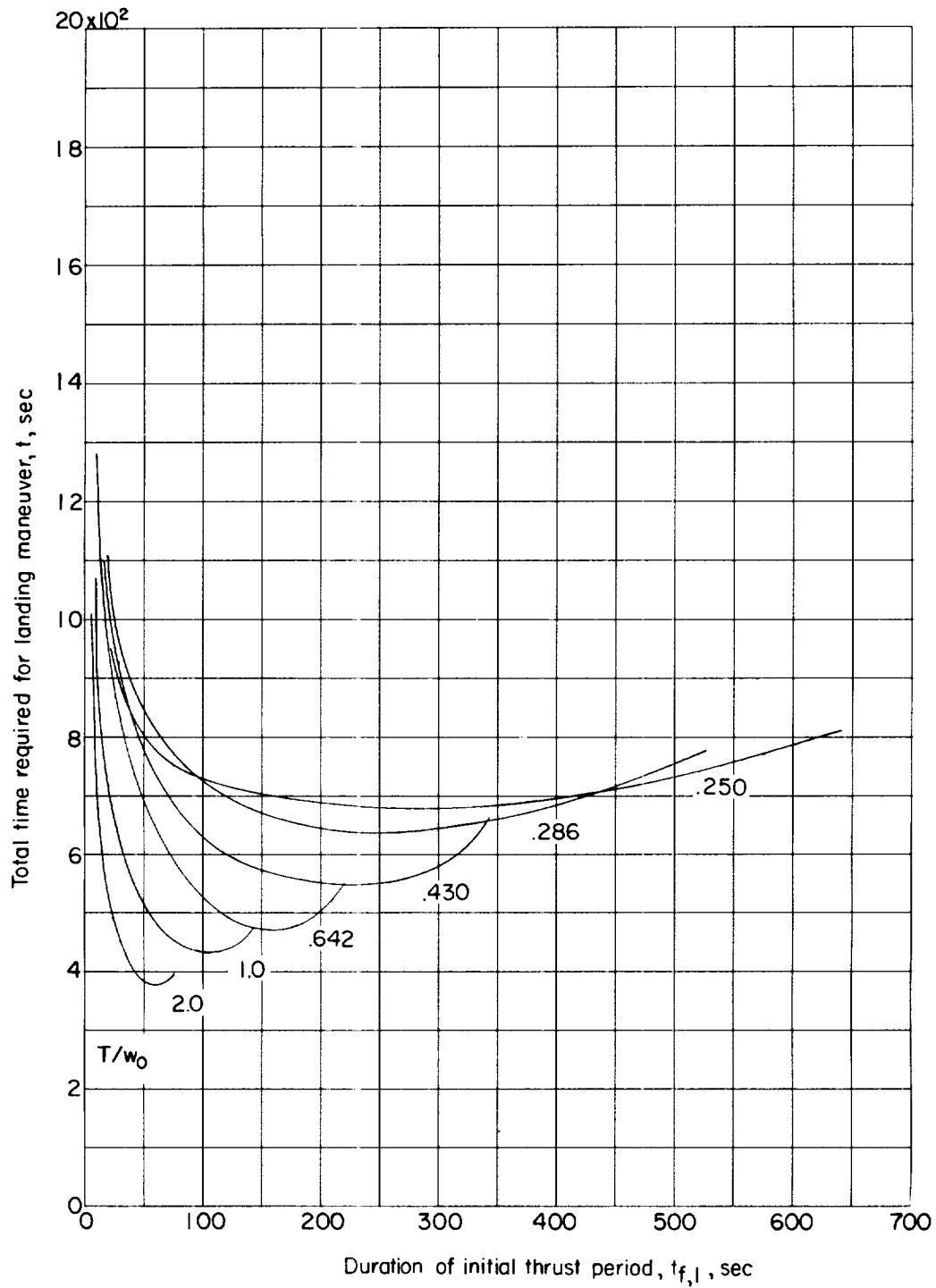


Figure 10.- Variation of total time required for landing as a function of T/w_0 and the duration of initial thrusting period.

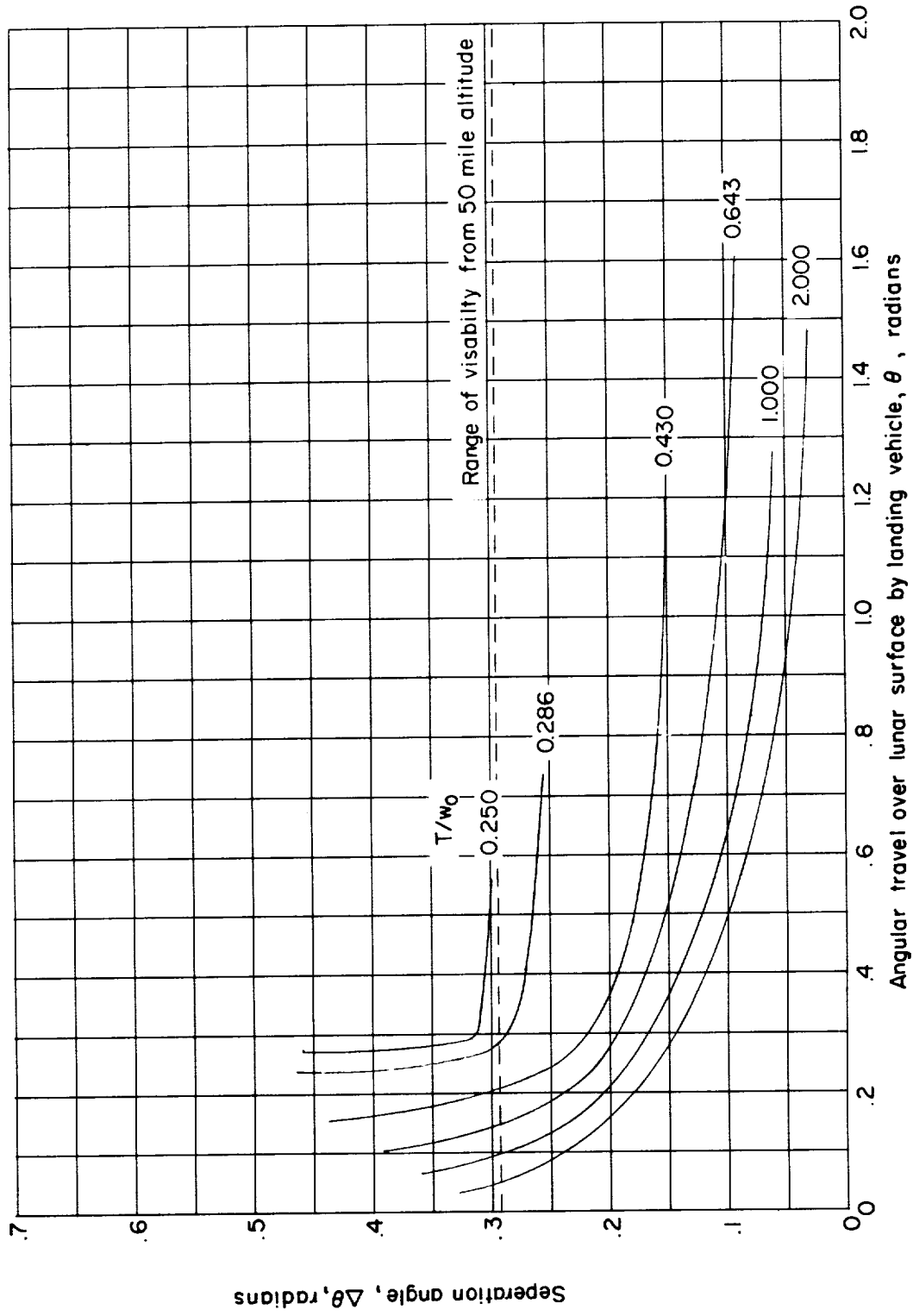
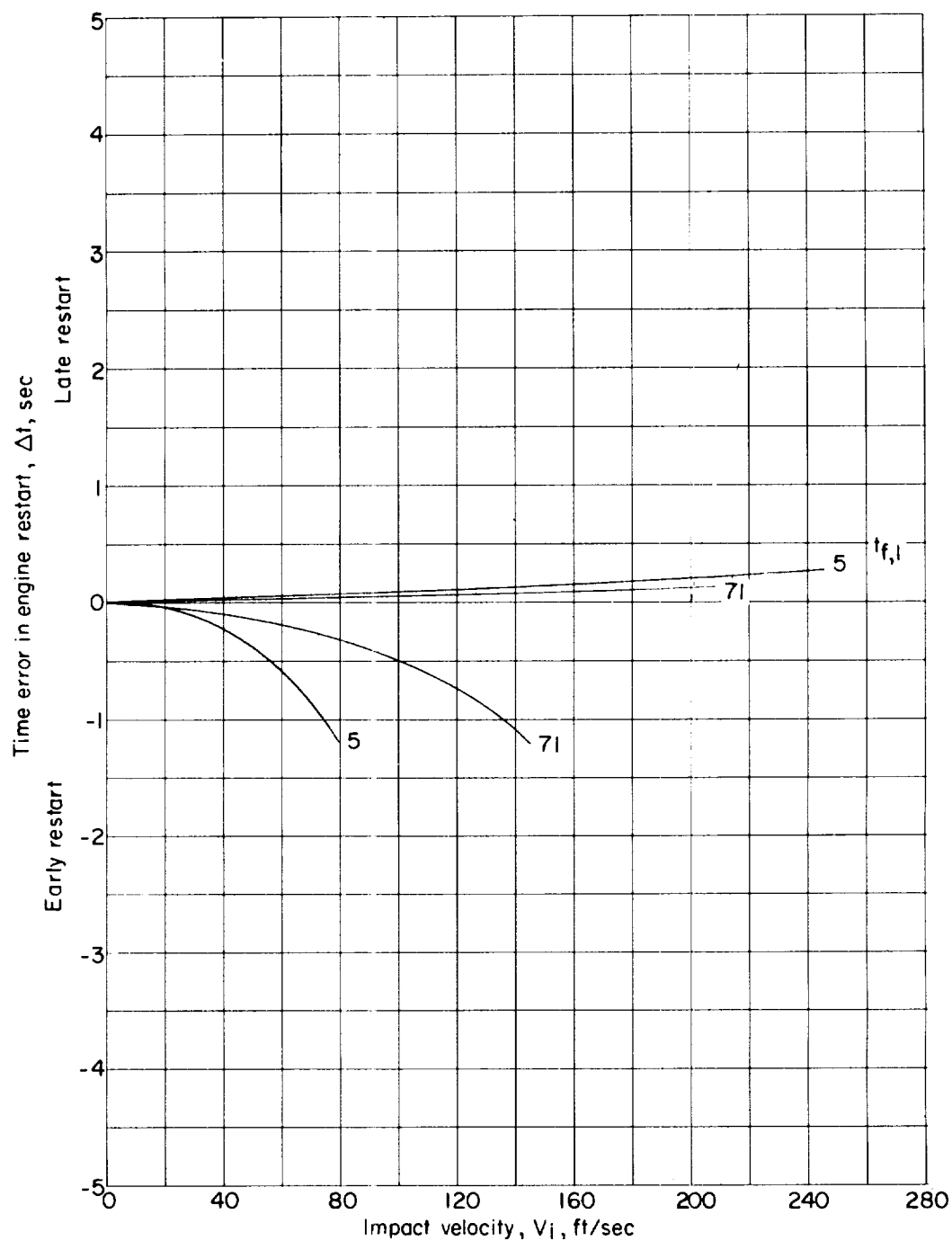
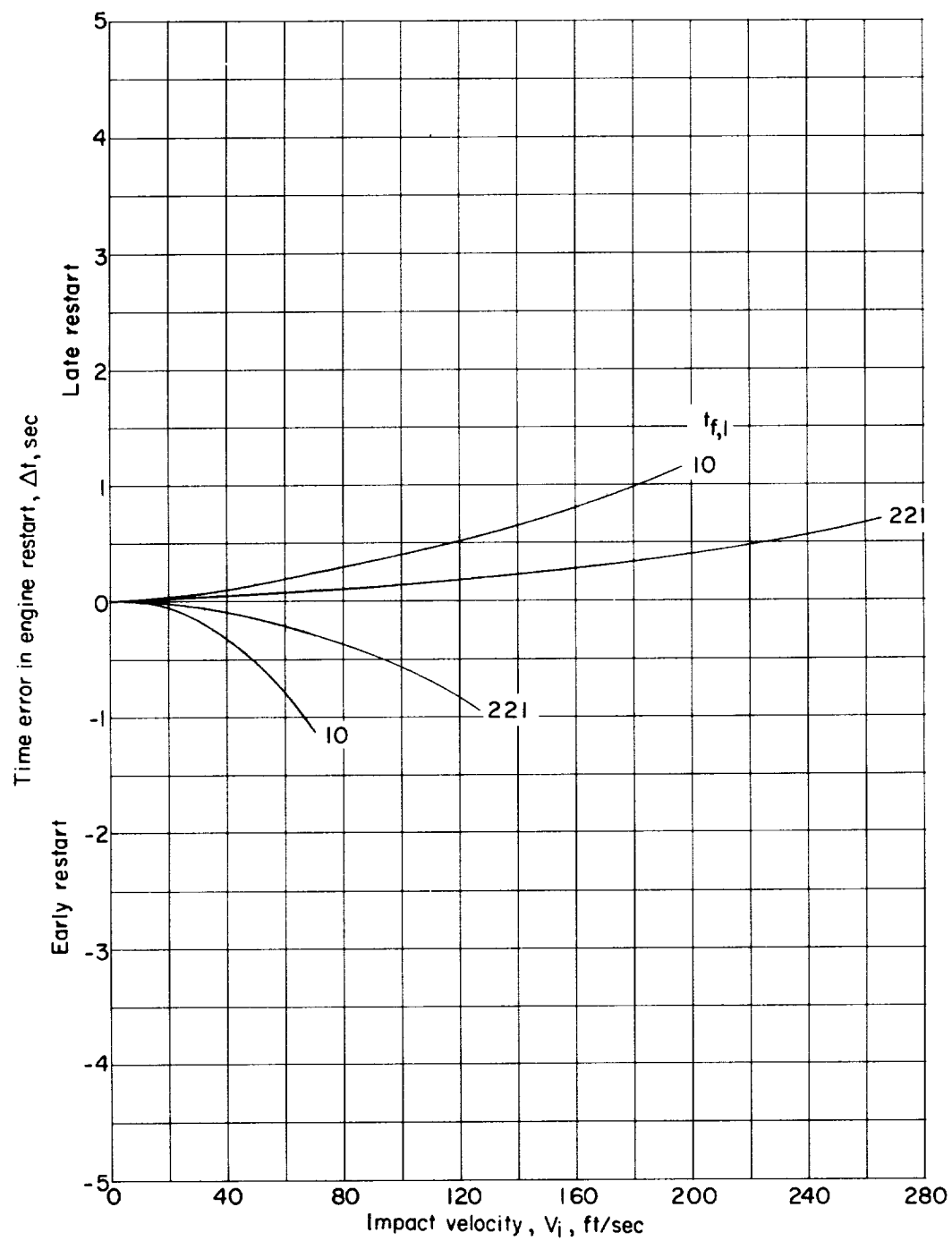


Figure 11.- Separation angle between orbiting space station and landing vehicle at touchdown of landing vehicle.



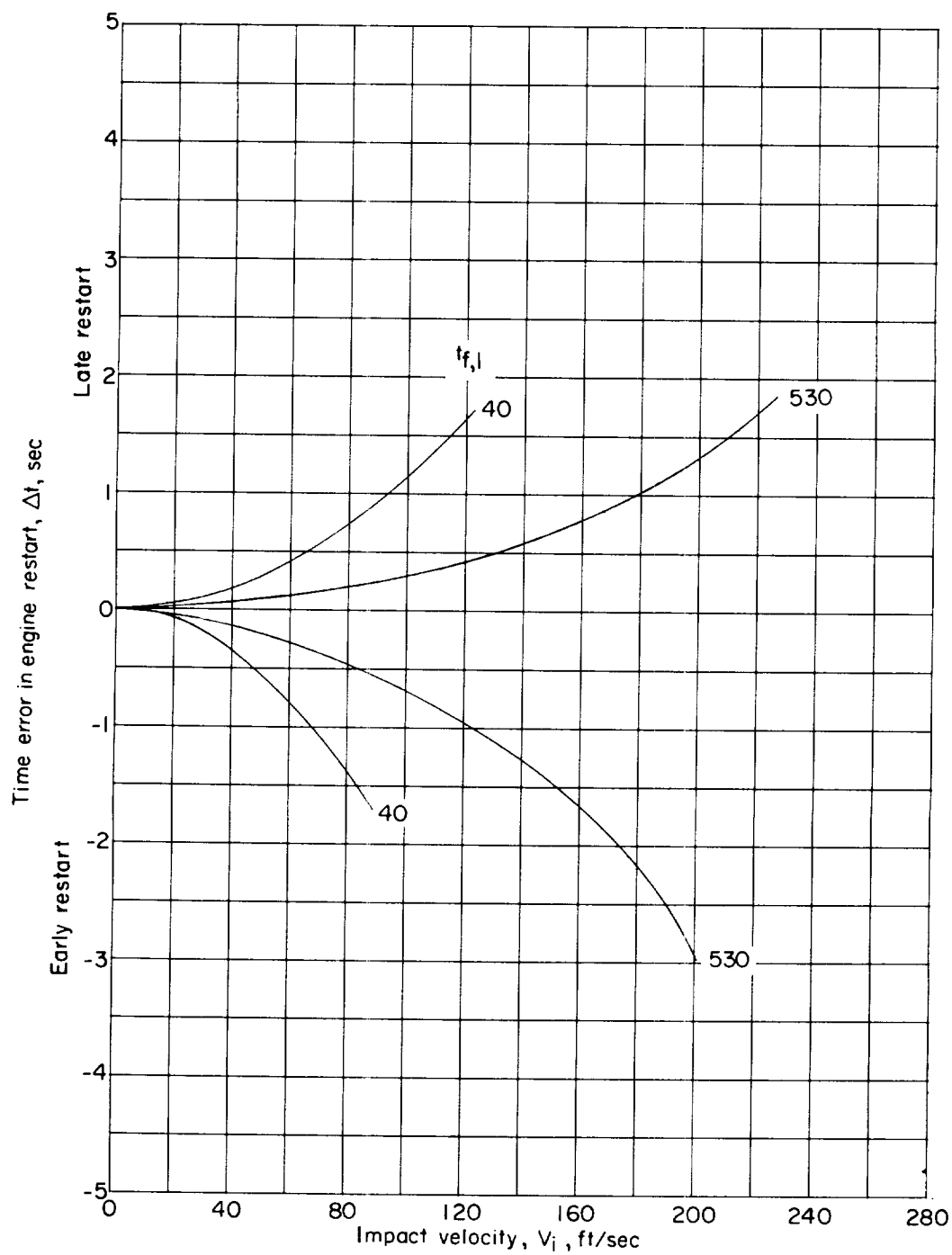
(a) $T/w_0 = 2.0$.

Figure 12.- Effect of timing error in engine restart on the impact velocity.



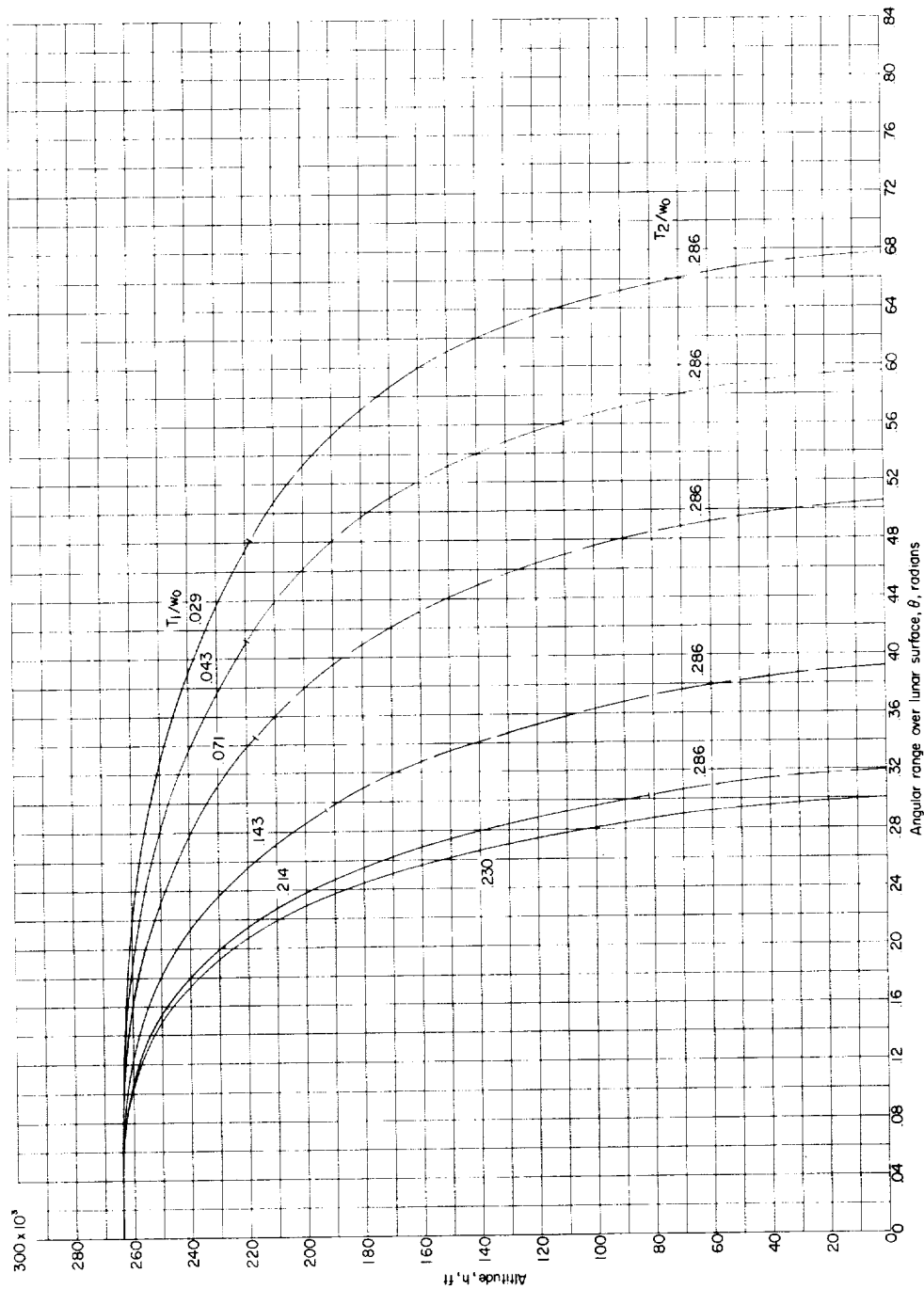
(b) $T/w_0 = 0.642$.

Figure 12.- Continued.



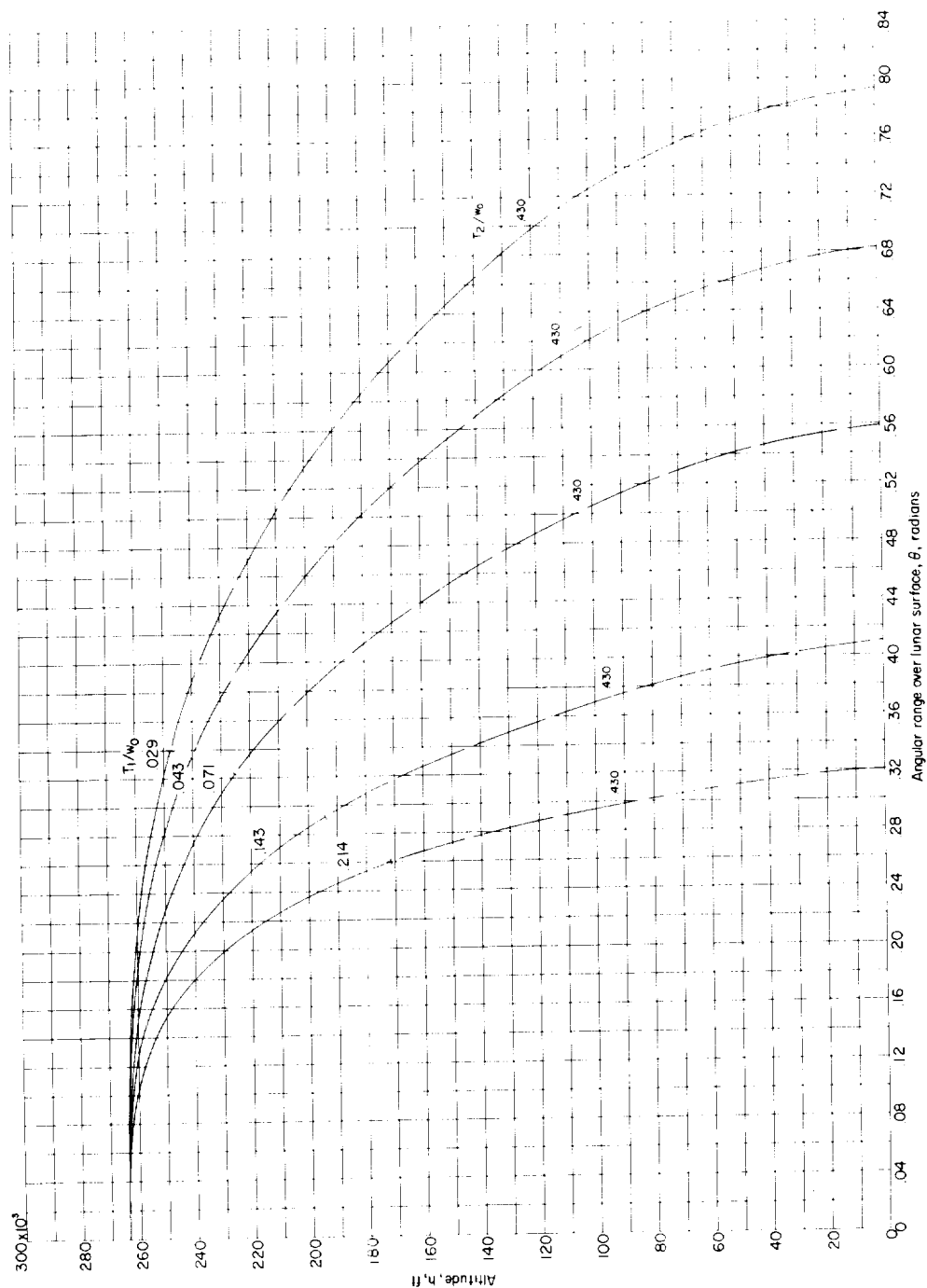
(c) $T/w_0 = 0.286$.

Figure 12.- Concluded.



(a) $T_2/w_0 = 0.286$.

Figure 13.- Trajectory characteristics. Variable-thrust landing mode.



(b) $T_2/w_0 = 0.430$.

Figure 13.- Concluded.

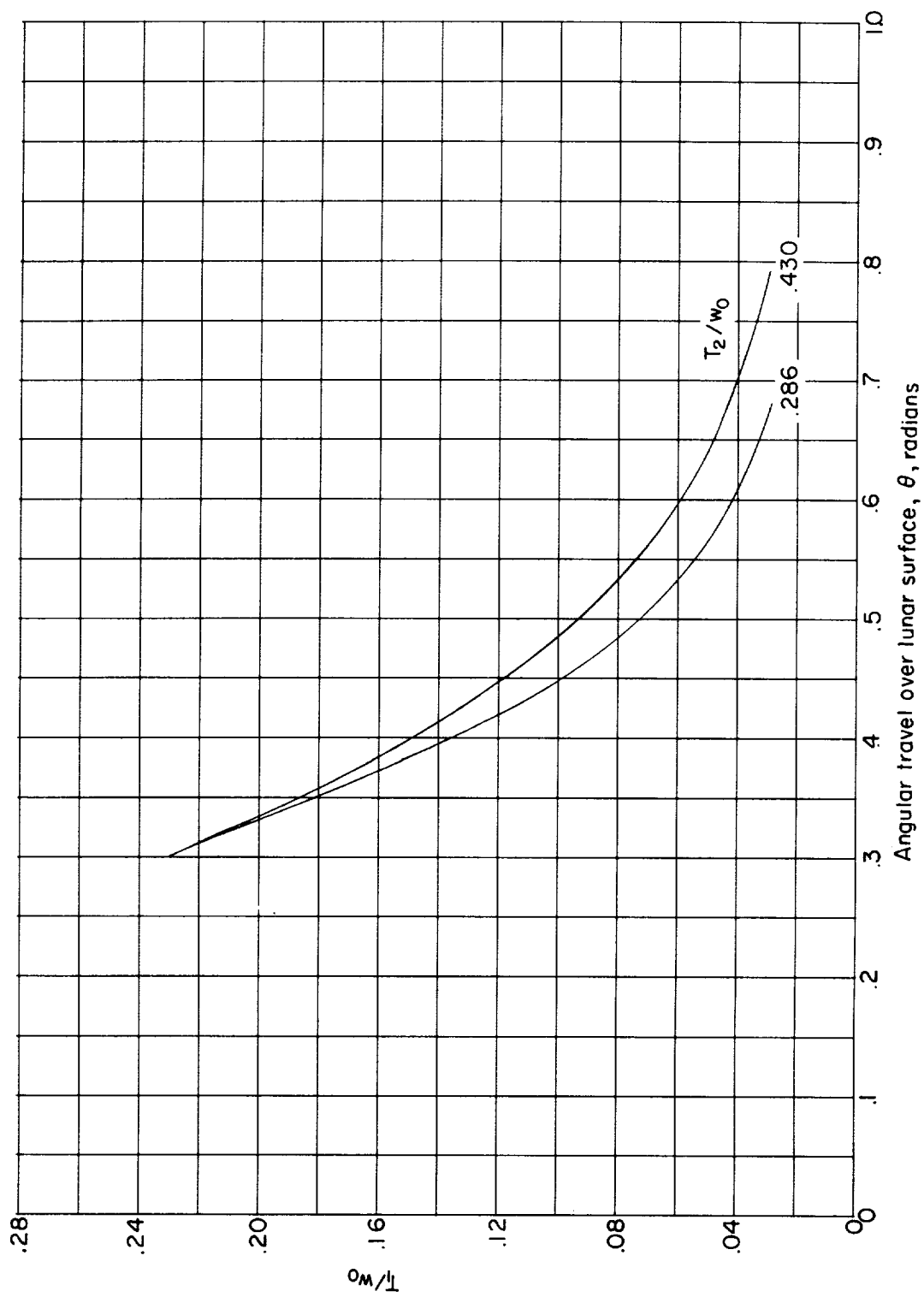
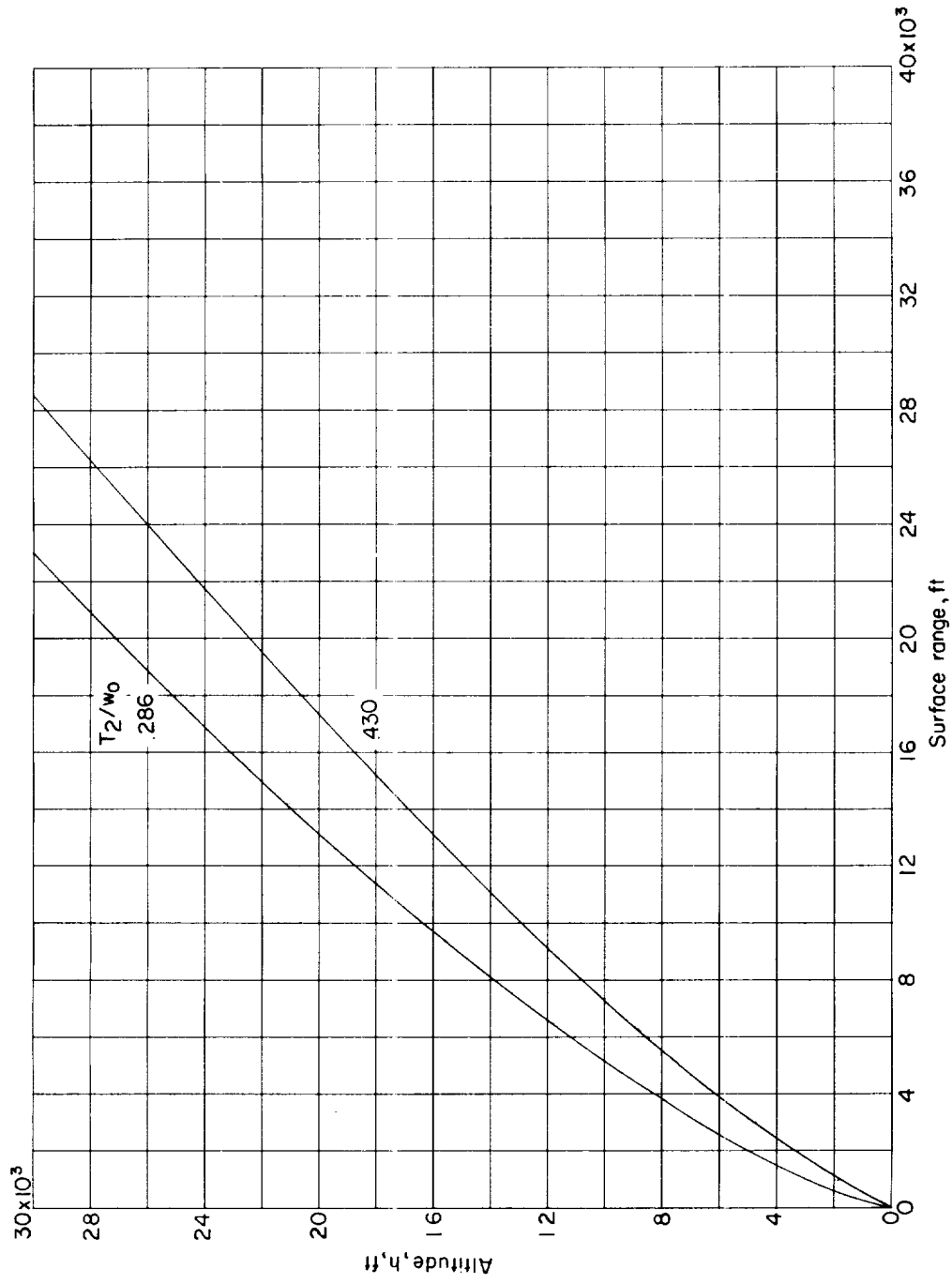
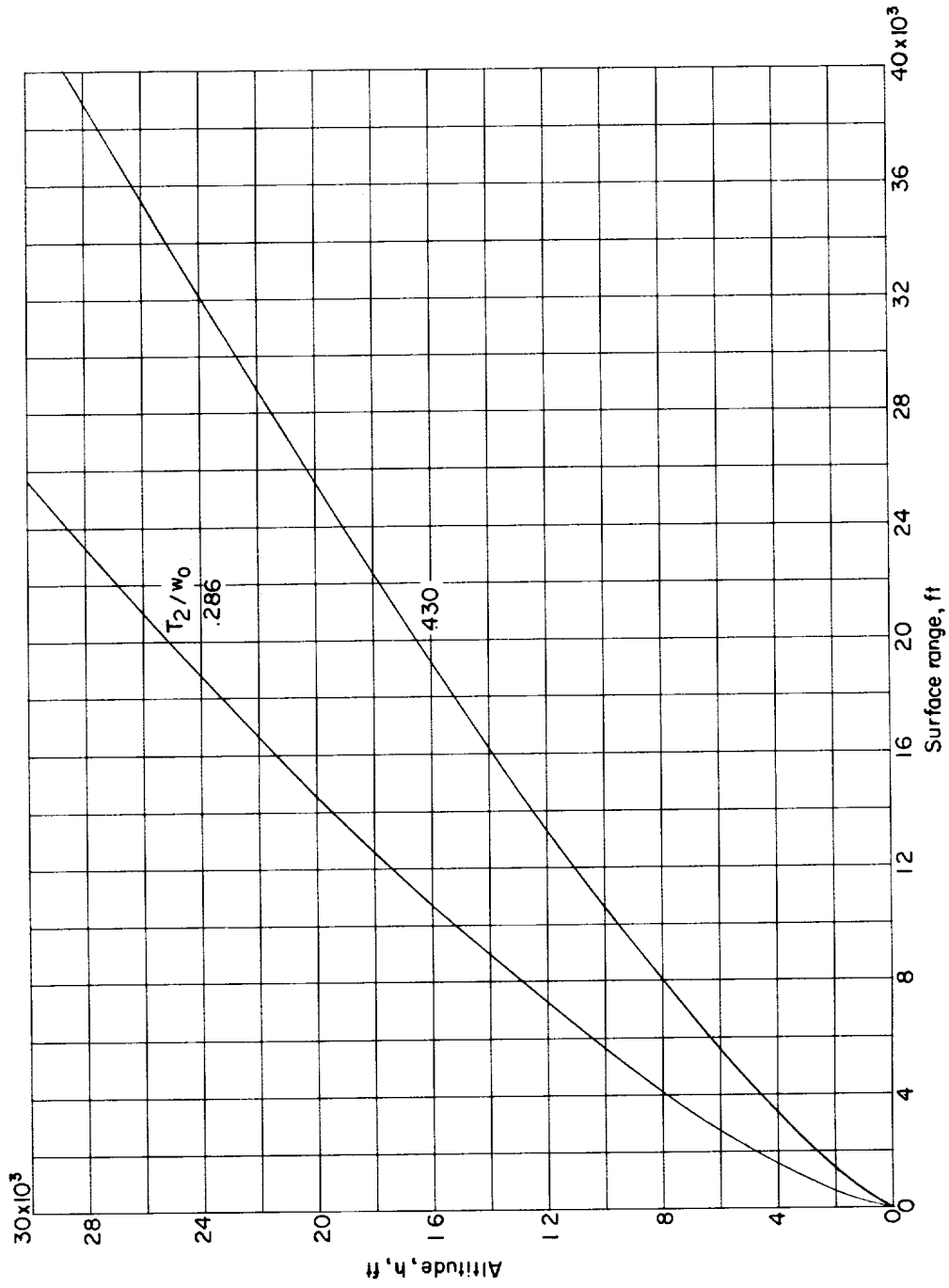


Figure 14.- Effect of initial thrust level on range. Variable-thrust mode.



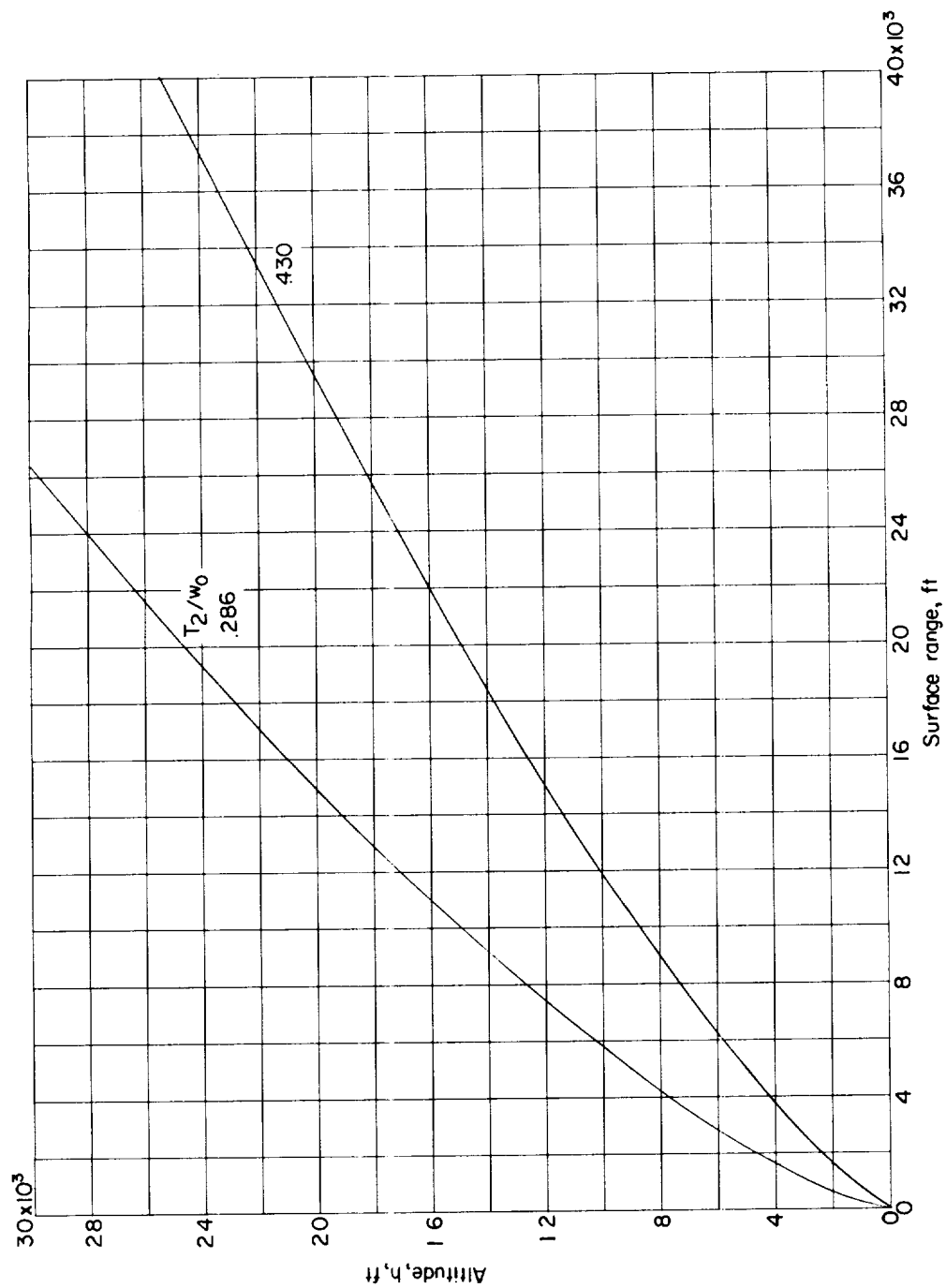
(a) $T_1/w_0 = 0.214$.

Figure 15.- Trajectory characteristics near touchdown point. Variable-thrust landing mode.



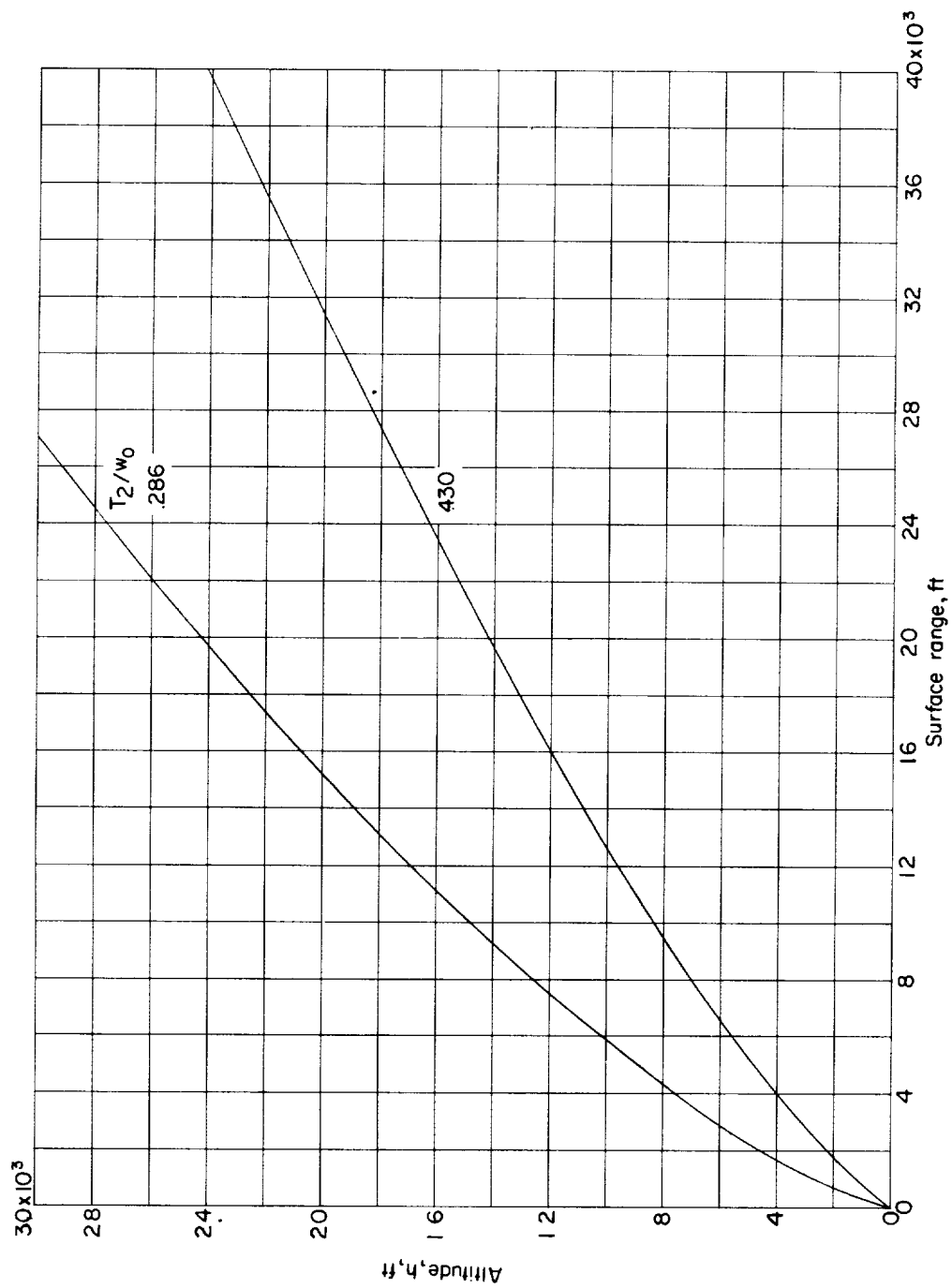
(b) $T_1/w_0 = 0.143$.

Figure 15.- Continued.



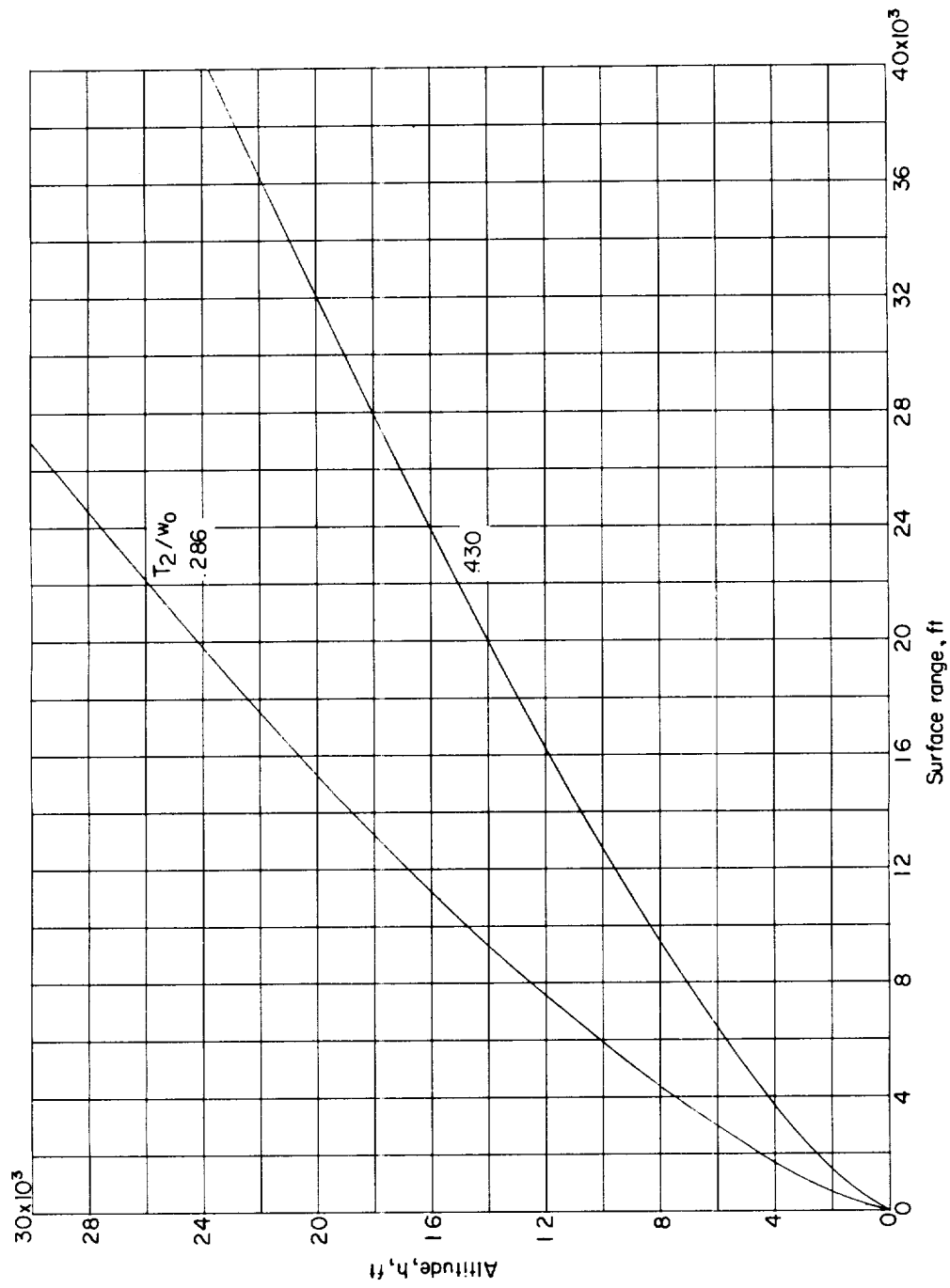
(c) $T_1/w_0 = 0.072$.

Figure 15.- Continued.



(d) $T_1/w_0 = 0.043$.

Figure 15.- Continued.



(e) $T_1/w_0 = 0.029$.

Figure 15.- Concluded.

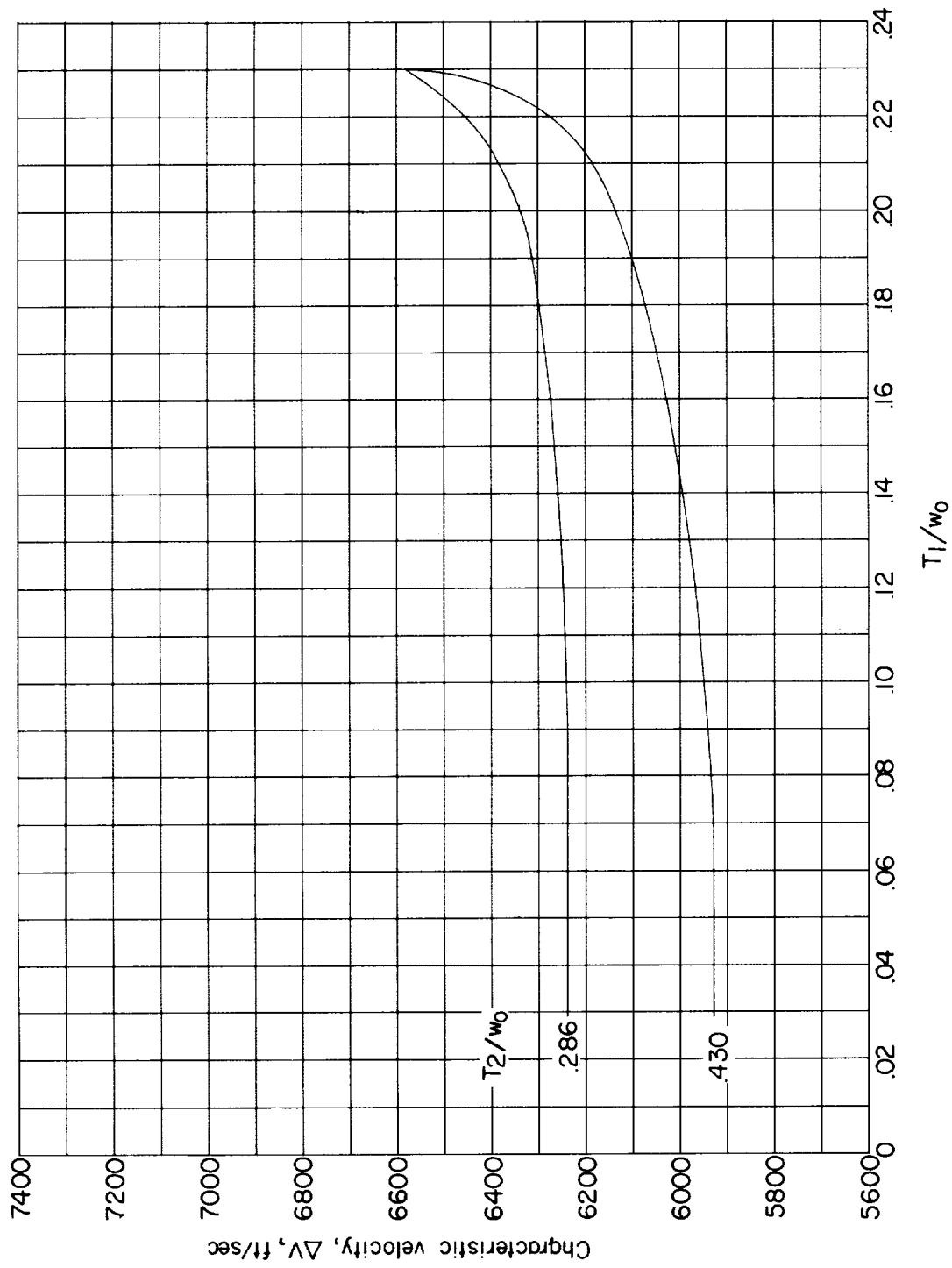


Figure 16.- Characteristic velocity as a function of T_1/w_0 . Variable-thrust landing mode.

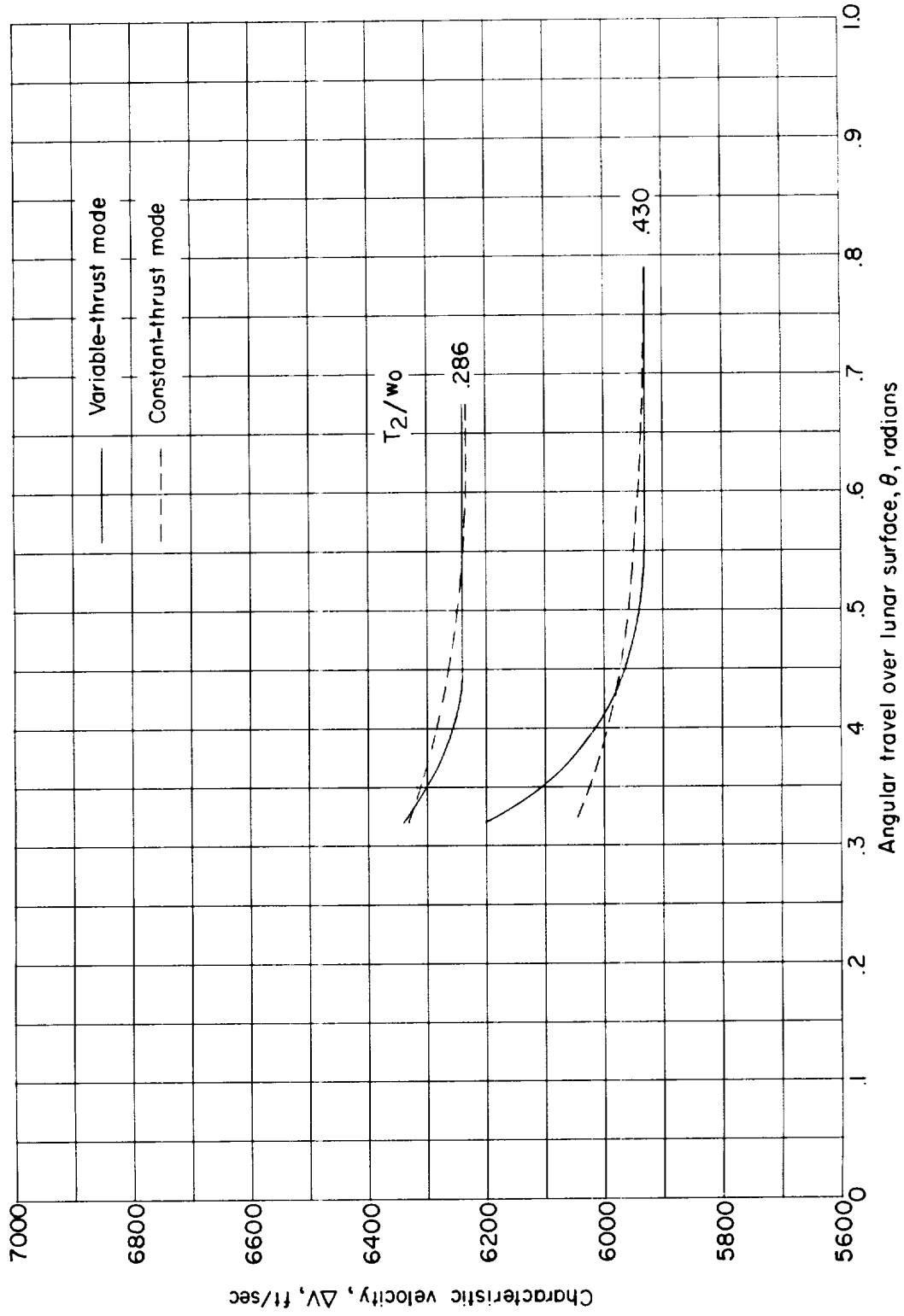


Figure 17.- Comparison of the characteristic velocity of the two thrust modes as a function of angular travel and T_2/w_0 .

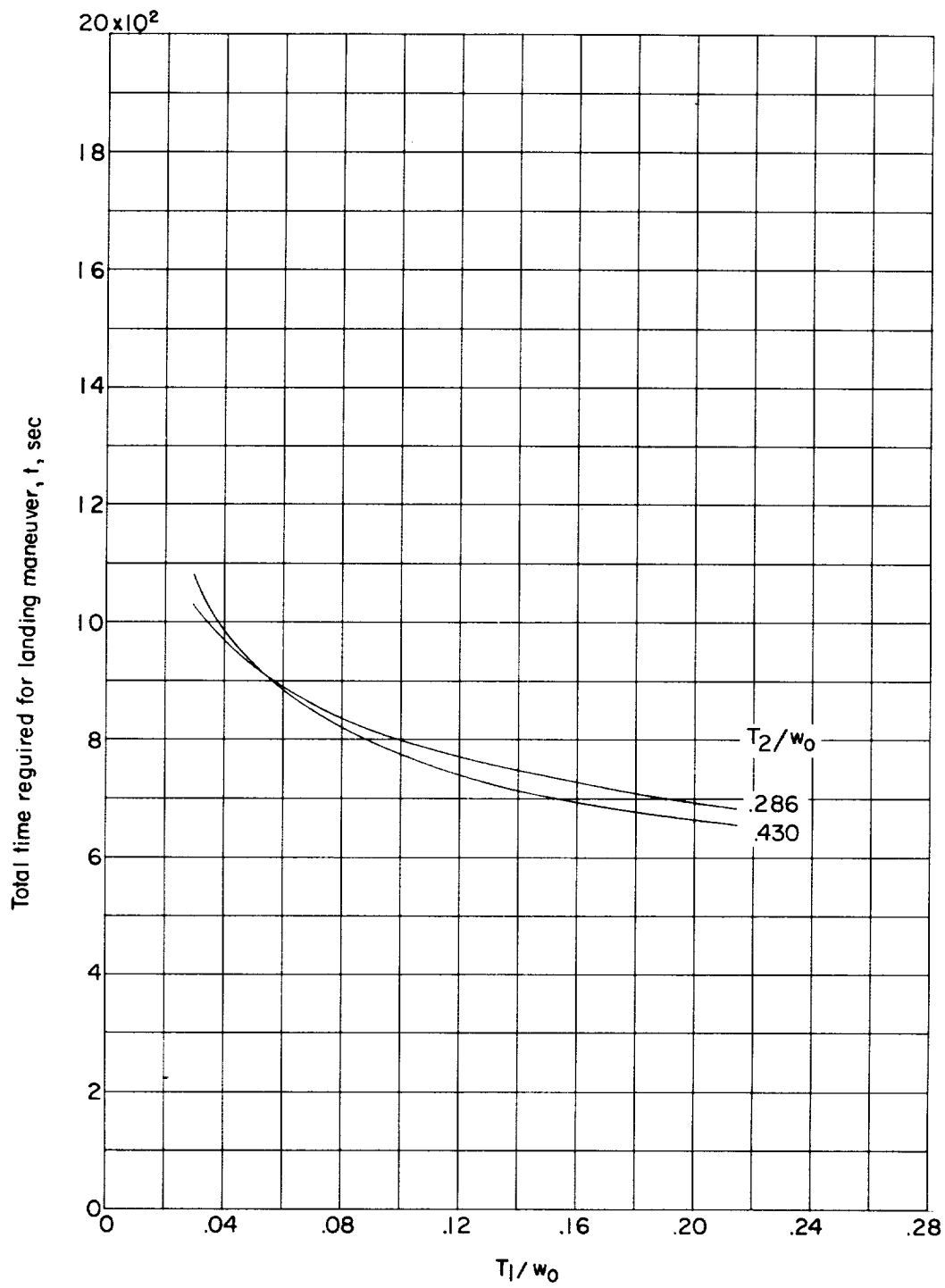


Figure 18.- Variation of total time required for landing as a function of T_1/w_0 and T_2/w_0 .

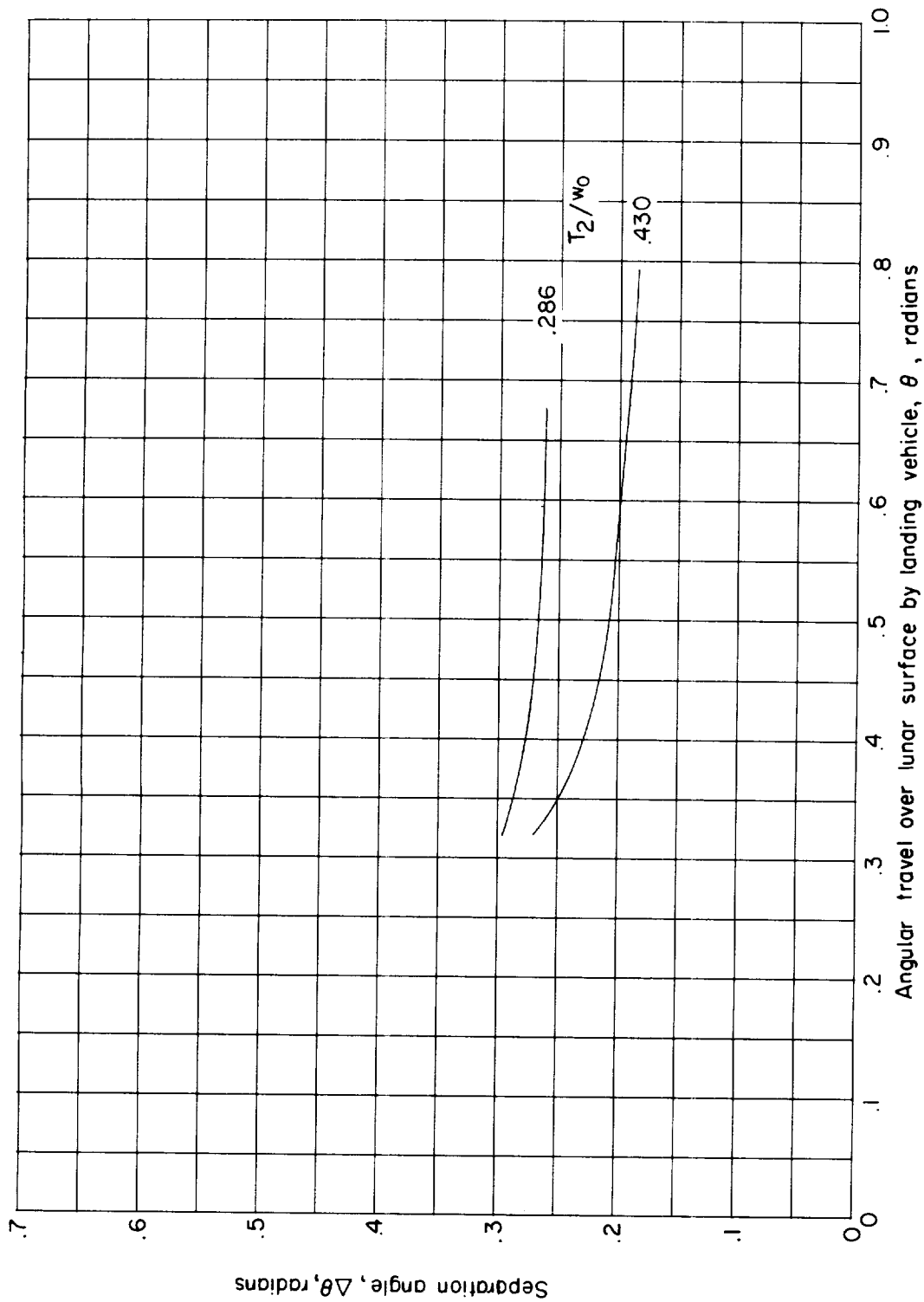


Figure 19.- Separation angle between orbiting space station and landing vehicle at touchdown of landing vehicle.

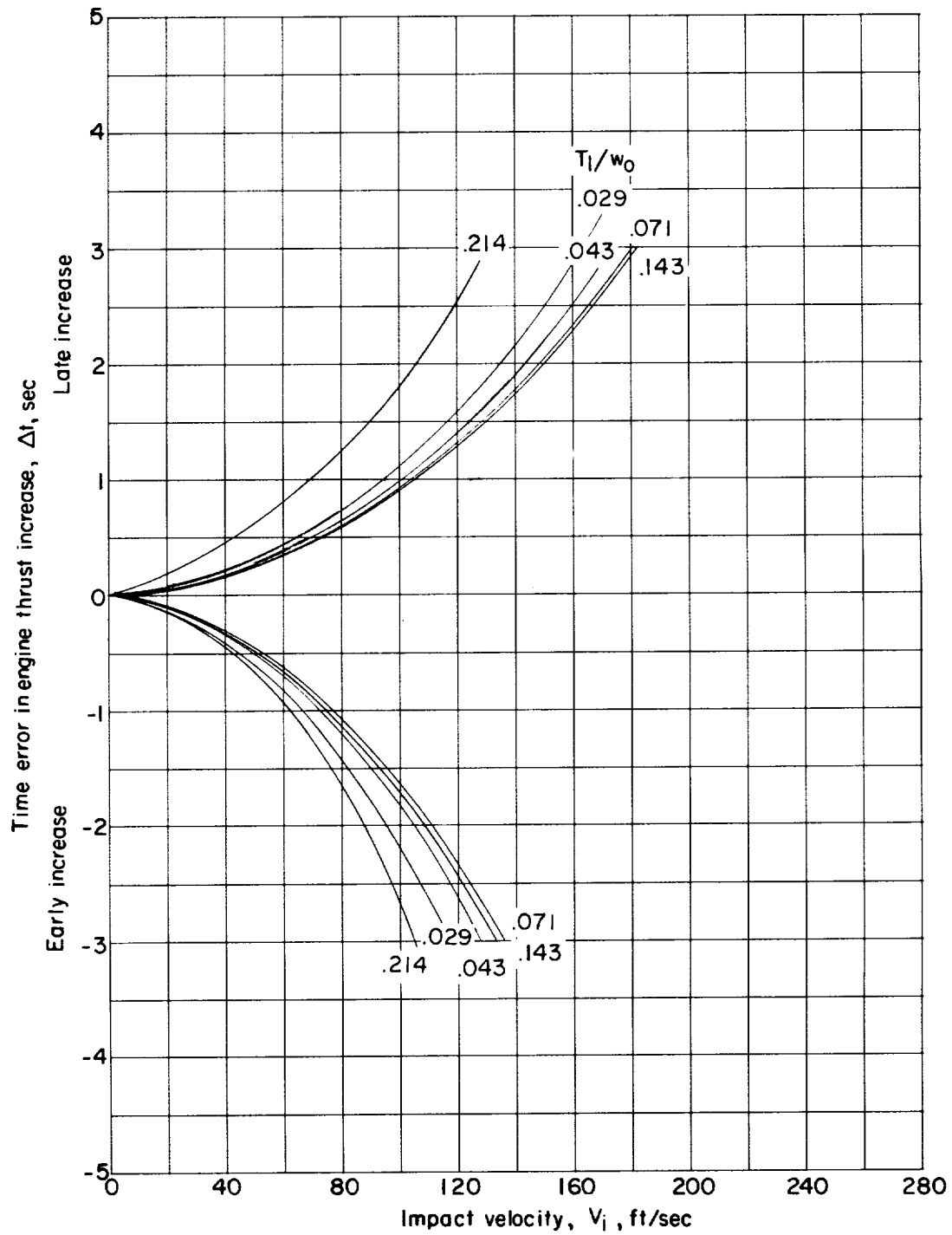


Figure 20.- Effect of timing error in engine increase in thrust level on impact velocity. $T_2/w_0 = 0.286$.

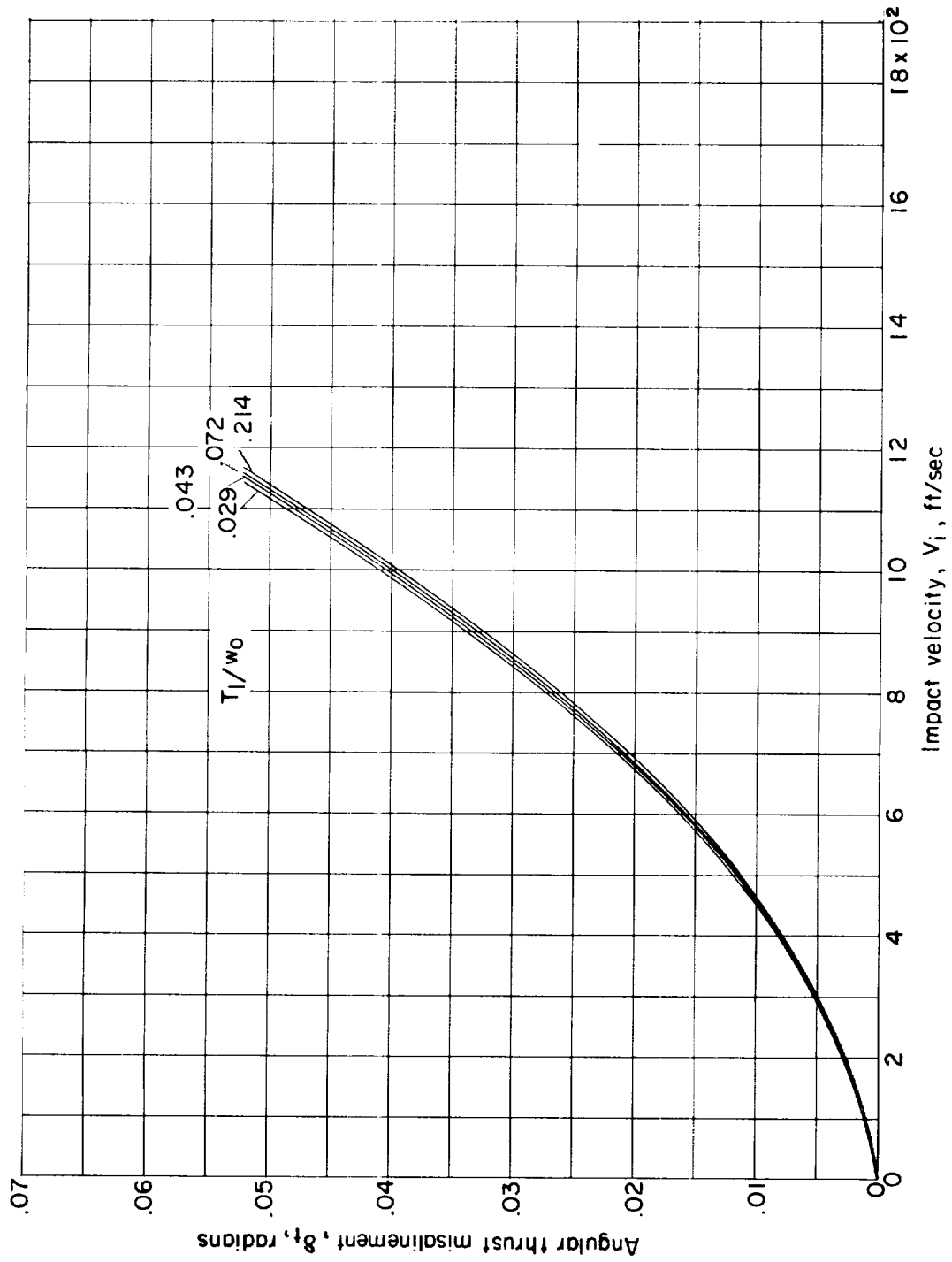


Figure 21.- Effect of angular thrust misalignment on the impact velocity. $T_2/w_0 = 0.286$.

